## 3.13 Solutions of exercises

**Solution of Exercise 3.3.2**

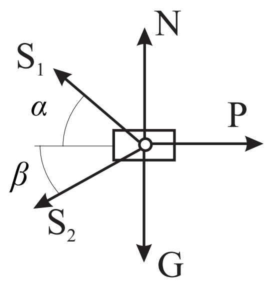


Figure 3.86: Exercise 3.3.2. Free-body diagram

Forces in springs:

$$S_1 = k_1(\sqrt{l_{01}^2 + x^2} - l_{01})$$

$$S_2 = k_2(\sqrt{l_{02}^2 + x^2} - l_{02})$$

Geometry:

$$\sin\alpha = \frac{l_{01}}{\sqrt{l_{01}^2 + x^2}}, \quad \cos\alpha = \frac{x}{\sqrt{l_{01}^2 + x^2}}$$

$$\sin\beta = \frac{l_{02}}{\sqrt{l_{02}^2 + x^2}}, \quad \cos\beta = \frac{x}{\sqrt{l_{02}^2 + x^2}}$$

Equations of equilibrium:

$$P - S_1\cos\alpha - S_2\cos\beta = 0$$

$$N + S_1\sin\alpha - G - S_2\sin\beta = 0$$

Solution:

$$P = S_1\cos\alpha + S_2\cos\beta$$

Result:

$$P = 77.6\ \mathrm{N}$$

*Notice:* The second equation of equlibrium is not necessary for finding the force $P$. It can be used for determination of the reaction force $N$.

**Solution of Exercise 3.3.3**

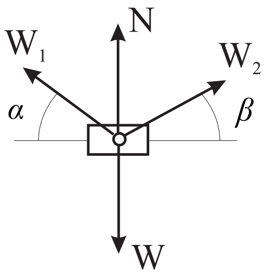


Figure 3.87: Exercise 3.3.3. Free-body diagram

Geometry:

$$\sin\alpha = \frac{h}{\sqrt{h^2+x^2}}, \quad \cos\alpha = \frac{x}{\sqrt{h^2+x^2}}$$

$$\sin\beta = \frac{h}{\sqrt{(l-x)^2+h^2}}, \quad \cos\beta = \frac{l-x}{\sqrt{(l-x)^2+h^2}}$$

Equations of equilibrium:

$$W_2\cos\beta - W_1\cos\alpha = 0$$

$$N + W_1\sin\alpha + W_2\sin\beta - W = 0$$

Solution:

$$W_2\frac{l-x}{\sqrt{(l-x)^2+h^2}} - W_1\frac{x}{\sqrt{h^2+x^2}} = 0 \quad => x_{eq}$$

*Notice:* See Matlab file S213.m for numerical solution.

**Solution of Exercise 3.3.4**

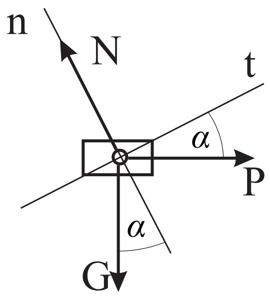


Figure 3.88: Exercise 3.3.4. Free-body diagram

Geometry:

$$y = \frac{b}{a^2}x^2$$

$$y' = \tan\alpha = \frac{2b}{a^2}x$$

Equations of equilibrium:

$$P\cos\alpha - G\sin\alpha = 0$$

$$N - G\cos\alpha - P\sin\alpha = 0$$

From equation of equilibrium we have:

$$\tan\alpha = \frac{P}{G}$$

Solution:

$$x_{eq} = \frac{a^2}{2b}\frac{P}{G}$$

Result:

$$x_{eq} = 0.036 \text{ m}$$

**Solution of Exercise 3.3.5**

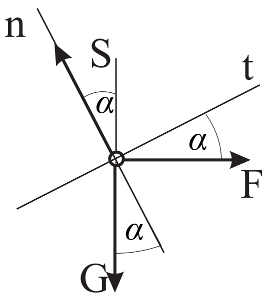


Figure 3.89: Exercise 3.3.5. Free-body diagram

Geometry:

$$\tan\alpha = \frac{x_{eq}}{\sqrt{l^2 - x_{eq}^2}}$$

Equations of equilibrium:

$$F\cos\alpha - G\sin\alpha = 0$$

$$S - G\cos\alpha - F\sin\alpha = 0$$

Solution:

$$\tan\alpha = \frac{x_{eq}}{\sqrt{l^2 - x_{eq}^2}} = \frac{F}{G} \qquad \rightarrow x_{eq}$$

*Notice:*See Matlab file S216.m for numerical solution.

**Solution of Exercise 3.4.2**

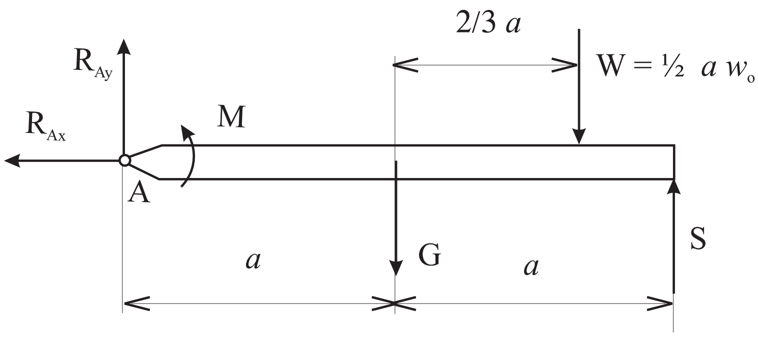


Figure 3.90: Exercise 3.4.2. Free-body diagram

Equations of equilibrium:

$$\begin{aligned}
\sum F_{ix}: &\quad R_{Ax} &= 0\\
\sum F_{iy}: &\quad R_{Ay}+S-W-G &= 0\\
\sum M_{iA}: &\quad -G\,a - W\tfrac{5}{3}\,a + S\,2\,a + M &= 0
\end{aligned}$$

Solution:

$$S=\frac{1}{2}\,G+\frac{5}{6}\,W-\frac{M}{2\,a}$$

$$l_0 = 0.5\,a+\xi = 0.5\,a+\frac{S}{k}$$

$$R_A = R_{Ay} = G+W-S$$

Result:

$$S = 603.4\ \mathrm{N}$$

$$l_0 = 0.112\ \mathrm{m}$$

$$R_A = 396.6\ \mathrm{N}$$

**Solution of Exercise 3.4.3**

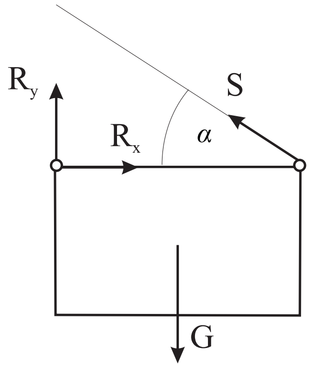


Figure 3.91: Exercise 3.4.3. Free-body diagram

Geometry:

$$\sin\alpha = \frac{a}{\sqrt{4\,a^2 + a^2}} = \frac{1}{\sqrt{5}}$$

Equations of equilibrium:

$$\begin{aligned} \sum F_{ix} :&\quad R_x \;-\; S\cos\alpha &= 0 \\ \sum F_{iy} :&\quad R_y \;+\; S\sin\alpha \;-\; G &= 0 \\ \sum M_{iA} :&\quad S\,\sin\alpha\,2\,a - G\,a &= 0 \end{aligned}$$

Solution:

$$S = \frac{G}{2\,\frac{1}{\sqrt{5}}} = \frac{\sqrt{5}}{2}\,G$$

$$k = \frac{S}{\xi} = \frac{S}{a\sqrt{5} - 2\,a} = \frac{167.7}{0.15\sqrt{5} - 0.3}$$

Result:

$$S = 167.7\ \mathrm{N}$$

$$k = 4737\ \mathrm{Nm}^{-1}$$

**Solution of Exercise 3.4.4**

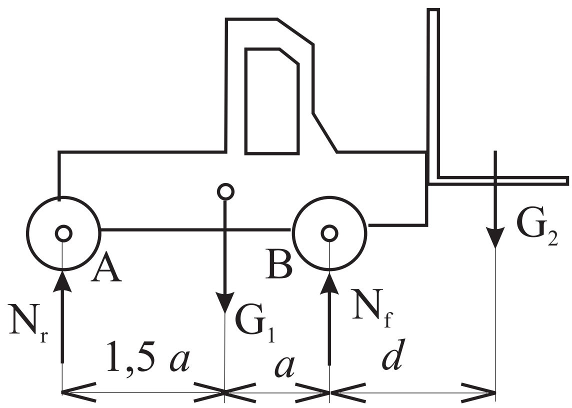


Figure 3.92: Exercise 3.4.4. Free-body diagram

Equations of equilibrium:

$$\begin{aligned} \sum \quad M_{iA}: \quad N_f\, 2.5\, a - G_1\, 1.5\, a - G_2\, (2.5\, a + d) &= 0 \\ \sum \quad M_{iB}: \quad G_1\, a - N_r\, 2.5\, a - G_2\, d &= 0 \end{aligned}$$

Solution:

$$N_f = \frac{1.5\, G_1\, a + (2.5\, a + d)\, G_2}{2.5\, a}$$

$$N_r = \frac{G_1\, a + G_2\, d}{2.5\, a}$$

Result:

$$N_f = 6700 \text{ N}$$

$$N_r = 300 \text{ N}$$

**Solution of Exercise 3.4.5**

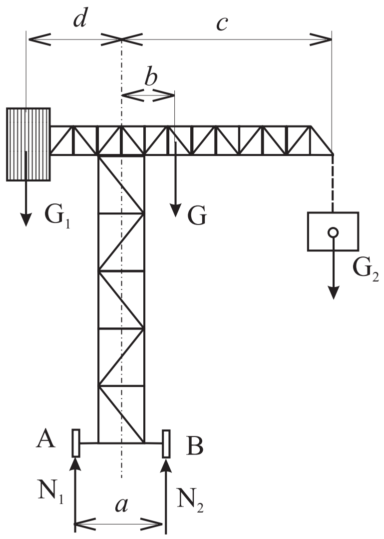


Figure 3.93: Exercise 3.4.5. Free-body diagram

a) Minimum counterweight $G_{1\min}$ for the crane not to lose its stability with $G_2 = 0$ . We suppose $N_1 = 0$ .

Equations of equilibrium:

$$\sum M_{i\mathrm{B}}: \quad G_{1\min}\,(d + \tfrac{a}{2}) - G\,(b - \tfrac{a}{2}) \;\; = \;\; 0$$

Solution:

$$G_{1\min} = \frac{b - \frac{a}{2}}{d_{+\frac{a}{2}}}\, G = 1538 \text{ N}$$

b) Maximum weight $G_{2\max} = 0$ for the crane not to loose its stability with $G_{1\max}$.

First we determine $G_{1\max}$ from the condition $N_2 = 0$, $G_2 = 0$. Equations of equilibrium:

$$\sum M_{i\mathrm{A}}: \quad G_{1\max}\,(d - \tfrac{a}{2}) - G(b + \tfrac{a}{2}) \;\; = \;\; 0$$

$$G_{1\max} = \frac{b + \frac{a}{2}}{d - \frac{a}{2}}\, G = 11428 \text{ N}$$

Now we find $G_{2\max}$ supposing $N_1 = 0$.
Equations of equilibrium:

$$\sum M_{i\mathrm{B}}: \quad G_{1\max}\,(d + \tfrac{a}{2}) - G\,(b - \tfrac{a}{2}) - G_{2\max}\,(c - \tfrac{a}{2}) \;\; = \;\; 0$$

Solution:

$$G_{2\max} = \frac{G_{1\max}\left(d + \frac{a}{2}\right) - G\left(b - \frac{a}{2}\right)}{c - \frac{a}{2}} = 3475\ \mathrm{N}$$

## Solution of Exercise 3.5.2

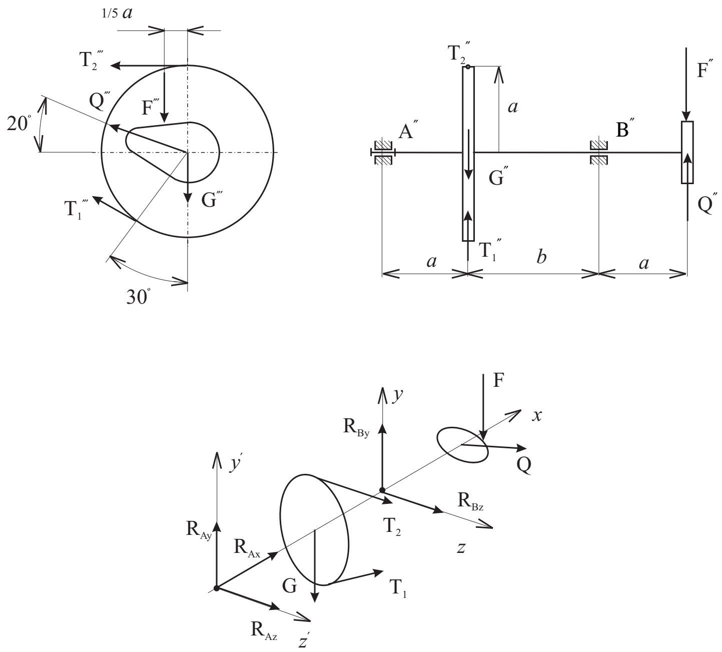


Figure 3.94: Exercise 3.5.2. Free-body diagram

Equations of equilibrium in a scalar form:

$$
\begin{aligned}
\sum F_{ix}: &\quad R_{\mathrm{A}x} &= 0\\
\sum M_{ix}: &\quad (T_2 - T_1)\,a + F\,0.2\,a &= 0\\
\sum M_{iy}: &\quad T_2\,b + R_{\mathrm{A}z}\,(a+b) + T_1\,\cos 30^\circ\,b - Q\,a\,\cos 20^\circ &= 0\\
\sum M_{iz}: &\quad -F\,a + Q\,a\,\sin 20^\circ - R_{\mathrm{A}y}\,(a+b) - T_1\,b\,\sin 30^\circ + G_1\,b &= 0\\
\sum M_{iy'}: &\quad -T_2\,a - T_1\,a\,\cos 30^\circ - R_{\mathrm{B}z}\,(a+b) - Q\,(a+b+a)\,\cos 20^\circ &= 0\\
\sum M_{iz'}: &\quad -G_1\,a + T_1\,a\,\sin 30^\circ + R_{\mathrm{B}y}\,(a+b) - F\,(a+b+a) + Q\,(a+b+a)\,\sin 20^\circ &= 0
\end{aligned}
$$

Equation of equilibrium in matrix form:

$$
\begin{bmatrix}
1 & -5 & 0 & 0 & 0 & 0\\
-a & a & 0 & 0 & 0 & 0\\
b\,\cos 30^\circ & b & 0 & a+b & 0 & 0\\
-b\,\sin 30^\circ & 0 & -a-b & 0 & 0 & 0\\
-a\,\cos 30^\circ & -a & 0 & 0 & 0 & -a-b\\
a\,\sin 30^\circ & 0 & 0 & 0 & a+b & 0
\end{bmatrix}
\begin{bmatrix}
T_1\\ T_2\\ R_{\mathrm{A}y}\\ R_{\mathrm{A}z}\\ R_{\mathrm{B}y}\\ R_{\mathrm{B}z}
\end{bmatrix}
=
$$

$$\begin{bmatrix} 0 \\ -F\,0.2\,a \\ Q\,a\,\cos 20^\circ \\ F\,a - Q\,a\,\sin 20^\circ - G_1\,b \\ Q\,(a+b+a)\,\cos 20^\circ \\ G_1\,a + F\,(a+b+a) - Q\,(a+b+a)\,\sin 20^\circ \end{bmatrix}$$

Result:
$R_{\mathrm{A}x} = 0$ N, $R_{\mathrm{A}y} = -946.2$ N, $R_{\mathrm{A}z} = -860$ N, $R_{\mathrm{A}} = 12786$ N, $R_{\mathrm{B}y} = 4349.4$ N, $R_{\mathrm{B}z} = -581.9$ N, $R_{\mathrm{B}} = 43881$ N, $T_1 = 10000$ N, $T_2 = 2000$ N

*Notice:* See Matlab file s3415.m for numerical solution.

**Solution of Exercise 3.5.3**

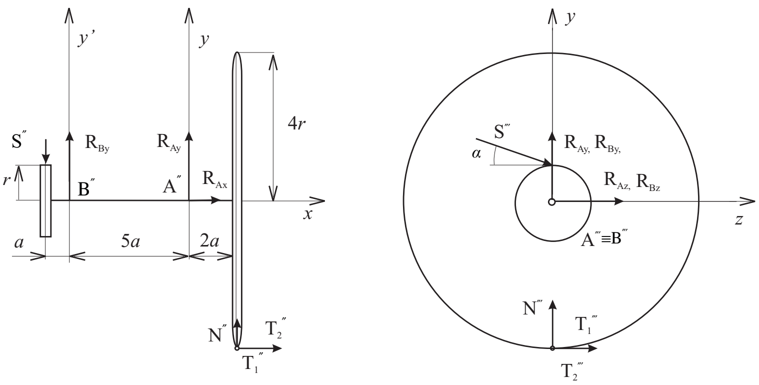


Figure 3.95: Exercise 3.5.3. Free-body diagram

Equations of equilibrium in scalar form:

$$\begin{array}{lrcl}
\sum F_{ix}: & R_{\mathrm{A}x} + T_2 & = & 0 \\
\sum M_{ix}: & S\cos\alpha\, r - T_1\, 4r & = & 0 \\
\sum M_{iy}: & R_{\mathrm{B}z}\, 5a - T_1\, 2a + S\cos\alpha\, 6a & = & 0 \\
\sum M_{iz}: & -R_{\mathrm{B}y}\, 5a + S\sin\alpha\, 6a + N\, 2a + T_2\, 4r & = & 0 \\
\sum M_{iy'}: & -R_{\mathrm{A}z}\, 5a + S\cos\alpha\, a - T_1\, 7a & = & 0 \\
\sum M_{iz'}: & R_{\mathrm{A}y}\, 5a + S\sin\alpha + N\, 7a + T_2\, 4r & = & 0
\end{array}$$

Equation of equilibrium in matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cos\alpha \\ 0 & 0 & 0 & 0 & 5 & 6\cos\alpha \\ 0 & 0 & 0 & -5 & 0 & 6\sin\alpha \\ 0 & 0 & -5 & 0 & 0 & \cos\alpha \\ 0 & 5 & 0 & 0 & 0 & \sin\alpha \end{bmatrix} \begin{bmatrix} R_{\mathrm{A}x} \\ R_{\mathrm{A}y} \\ R_{\mathrm{A}z} \\ R_{\mathrm{B}y} \\ R_{\mathrm{B}z} \\ S \end{bmatrix} = \begin{bmatrix} -T_2 \\ 4T_1 \\ 2T_1 \\ -2N - 4.08T_2 \\ 7T_1 \\ -7N - 4.08T_2 \end{bmatrix}$$

*Notice:* See Matlab file `S3421.m` for numerical solution.

**Solution of Exercise 3.5.4**

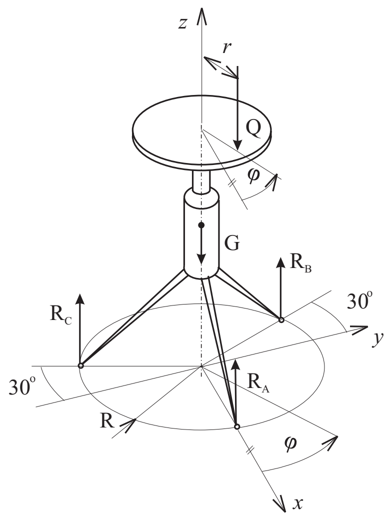


Figure 3.96: Exercise 3.5.4. Free-body diagram

Equations of equilibrium:

$$\begin{aligned}
\sum F_{iz}: &\quad -Q - G + R_\mathrm{A} + R_\mathrm{B} + R_\mathrm{C} &= 0\\
\sum M_{ix}: &\quad -Q\,r\,\sin\varphi + R_\mathrm{B}\,R\,\cos 30^\circ - R_\mathrm{C}\,R\,\cos 30^\circ &= 0\\
\sum M_{iy}: &\quad Q\,r\,\cos\varphi - R_\mathrm{A}\,R + R_\mathrm{B}\,R\,\sin 30^\circ + R_\mathrm{C}\,R\,\sin 30^\circ &= 0
\end{aligned}$$

Solution:

$$R_\mathrm{A} = Q + G - (R_\mathrm{B} + R_\mathrm{C})$$

$$R_\mathrm{B} - R_\mathrm{C} = \frac{Q\,r\,\sin\varphi}{R\cos 30^\circ} = f_1(\varphi)$$

$$R_\mathrm{B} + R_\mathrm{C} = \frac{(Q+G)\,R - Q\,r\cos\varphi}{R\,(1+\sin 30^\circ)} = f_2(\varphi)$$

$$R_\mathrm{B} = \frac{f_1(\varphi) + f_2(\varphi)}{2}$$

$$R_\mathrm{C} = \frac{f_2(\varphi) - f_1(\varphi)}{2}$$

*Notice:* See Matlab file `S3423.m` for numerical solution.

**Solution of Exercise 3.5.5**

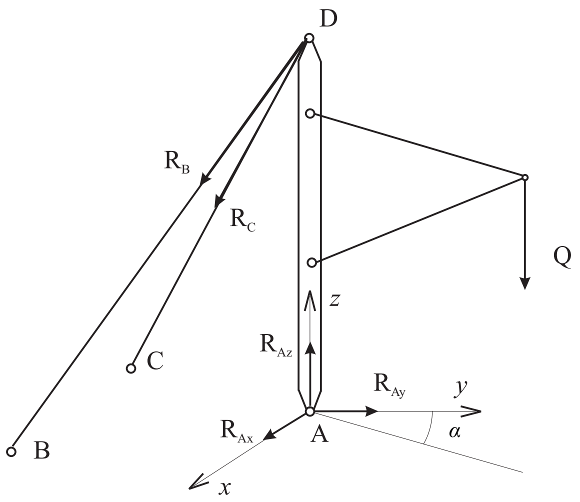


Figure 3.97: Exercise 3.5.5. Free-body diagram

Equations of equilibrium:

$$\begin{aligned}
\sum F_{ix}: &\quad R_{\mathrm{A}x} + R_{\mathrm{B}x} + R_{\mathrm{C}x} &= 0\\
\sum F_{iy}: &\quad R_{\mathrm{A}y} + R_{\mathrm{B}y} + R_{\mathrm{C}y} &= 0\\
\sum F_{ix}: &\quad -Q + R_{\mathrm{A}z} + R_{\mathrm{B}z} + R_{\mathrm{C}z} &= 0\\
\sum M_{ix}: &\quad -3\,a\,Q\,\cos\alpha - 5\,a\,R_{\mathrm{B}y} - 5\,a\,R_{\mathrm{C}y} &= 0\\
\sum M_{iy}: &\quad 3\,a\,Q\,\sin\alpha + 5\,a\,R_{\mathrm{B}x} + 5\,a\,R_{\mathrm{C}x} &= 0
\end{aligned}$$

Geometry:

$$\cos\alpha_{\mathrm{B}} = \frac{x_{\mathrm{B}} - x_{\mathrm{D}}}{\mathrm{BD}}, \quad \cos\beta_{\mathrm{B}} = \frac{y_{\mathrm{B}} - y_{\mathrm{D}}}{\mathrm{BD}}, \quad \cos\gamma_{\mathrm{B}} = \frac{z_{\mathrm{B}} - z_{\mathrm{D}}}{\mathrm{BD}}$$

$$\cos\alpha_{\mathrm{C}} = \frac{x_{\mathrm{C}} - x_{\mathrm{D}}}{\mathrm{CD}}, \quad \cos\beta_{\mathrm{C}} = \frac{y_{\mathrm{C}} - y_{\mathrm{D}}}{\mathrm{CD}}, \quad \cos\gamma_{\mathrm{C}} = \frac{z_{\mathrm{C}} - z_{\mathrm{D}}}{\mathrm{CD}}$$

Components of reaction forces:

$$R_{\mathrm{B}x} = R_{\mathrm{B}}\cos\alpha_{\mathrm{B}}, \quad R_{\mathrm{B}y} = R_{\mathrm{B}}\cos\beta_{\mathrm{B}}, \quad R_{\mathrm{B}z} = R_{\mathrm{B}}\cos\gamma_{\mathrm{B}}$$

$$R_{\mathrm{C}x} = R_{\mathrm{C}}\cos\alpha_{\mathrm{C}}, \quad R_{\mathrm{C}y} = R_{\mathrm{C}}\cos\beta_{\mathrm{C}}, \quad R_{\mathrm{C}z} = R_{\mathrm{C}}\cos\gamma_{\mathrm{C}}$$

Equations of equilibrium in matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & \cos\alpha_{\mathrm{B}} & \cos\alpha_{\mathrm{C}} \\ 0 & 1 & 0 & \cos\beta_{\mathrm{B}} & \cos\beta_{\mathrm{C}} \\ 0 & 0 & 1 & \cos\gamma_{\mathrm{B}} & \cos\gamma_{\mathrm{C}} \\ 0 & 0 & 0 & -5\,\cos\beta_{\mathrm{B}} & -5\,\cos\beta_{\mathrm{C}} \\ 0 & 0 & 0 & 5\,\cos\alpha_{\mathrm{B}} & 5\,\cos\beta_{\mathrm{C}} \end{bmatrix} \begin{bmatrix} R_{\mathrm{A}x} \\ R_{\mathrm{A}y} \\ R_{\mathrm{A}z} \\ R_{\mathrm{B}} \\ R_{\mathrm{C}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ Q \\ -3\,Q\,\cos\alpha \\ -3\,Q\,\sin\alpha \end{bmatrix}$$

*Notice:* See Matlab file `S3410.m` for numerical solution.

**Solution of Exercise 3.5.6**

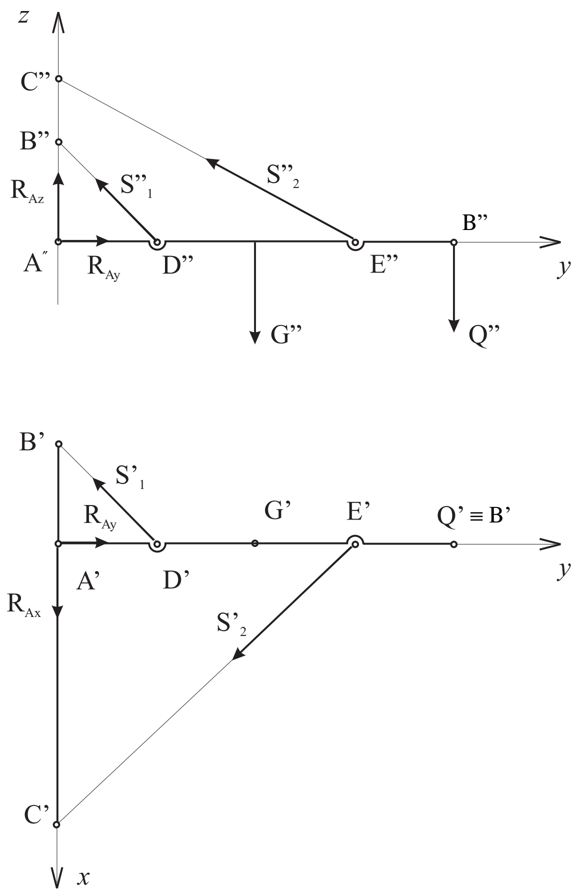


Figure 3.98: Exercise 3.5.6. Free-body diagram

Equations of equilibrium:

$$
\begin{aligned}
\sum F_{ix}: &\quad R_{Ax} + S_{1x} + S_{2x} &= 0\\
\sum F_{iz}: &\quad R_{Ay} + S_{1y} + S_{2y} &= 0\\
\sum F_{iy}: &\quad R_{Az} + S_{1z} + S_{2z} - Q - G &= 0\\
\sum M_{ix}: &\quad -4\,a\,Q + a\,S_{1z} + 3\,a\,S_{2z} - 2\,a\,G &= 0\\
\sum M_{iz}: &\quad -a\,S_{1x} - 3\,a\,S_{2x} &= 0
\end{aligned}
$$

Geometry:

$$
\cos\alpha_{DB} = \frac{x_B - x_D}{DB}, \quad \cos\beta_{DB} = \frac{y_B - y_D}{DB}, \quad \cos\gamma_{DB} = \frac{z_B - z_D}{DB}
$$

$$
\cos\alpha_{EC} = \frac{x_C - x_E}{EC}, \quad \cos\beta_{EC} = \frac{y_C - y_E}{EC}, \quad \cos\gamma_{EC} = \frac{z_C - z_E}{EC}
$$

Components of forces:

$$S_{1x} = S_1 \cos \alpha_{\rm DB}, \quad S_{1y} = S_1 \cos \beta_{\rm DB}, \quad S_{1z} = S_1 \cos \gamma_{\rm DB}$$

$$S_{2x} = S_2 \cos \alpha_{\rm EC}, \quad S_{2y} = S_2 \cos \beta_{\rm EC}, \quad S_{2z} = S_2 \cos \gamma_{\rm EC}$$

Equations of equilibrium in matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & \cos \alpha_{\rm DB} & \cos \alpha_{\rm EC} \\ 0 & 1 & 0 & \cos \beta_{\rm DB} & \cos \beta_{\rm EC} \\ 0 & 0 & 1 & \cos \gamma_{\rm DB} & \cos \gamma_{\rm EC} \\ 0 & 0 & 0 & \cos \gamma_{\rm DB} & 3 \cos \gamma_{\rm EC} \\ 0 & 0 & 0 & \cos \alpha_{\rm DB} & 3 \cos \alpha_{\rm EC} \end{bmatrix} \begin{bmatrix} R_{{\rm A}x} \\ R_{{\rm A}y} \\ R_{{\rm A}z} \\ S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ G + Q \\ 2\,G + 4\,Q \\ 0 \end{bmatrix}$$

*Notice:* See Matlab file `S3411.m` for numerical solution.

**Solution of Exercise 3.5.7**

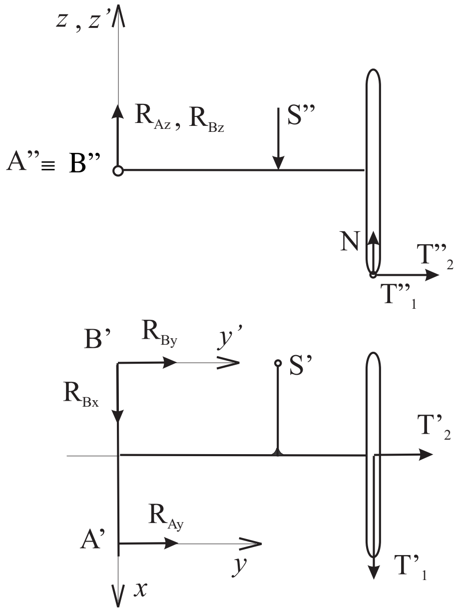


Figure 3.99: Exercise 3.5.7. Free-body diagram

Equations of equilibrium:

$$\begin{array}{lrcl}
\sum F_{ix}: & R_{\mathrm{B}x} + T_1 & = & 0 \\
\sum M_{ix}: & -2\,a\,S + 3\,a\,N + r\,T_2 & = & 0 \\
\sum M_{iy}: & a\,N - r\,T_1 - 2\,a\,S + 2\,a\,R_{\mathrm{B}z} & = & 0 \\
\sum M_{iz}: & -3\,a\,T_1 - a\,T_2 - 2\,a\,R_{\mathrm{B}y} & = & 0 \\
\sum M_{iy'}: & -a\,N - r\,T_1 - 2\,a\,R_{\mathrm{A}z} & = & 0 \\
\sum M_{iz'}: & a\,T_2 - 3\,a\,T_1 + 2\,a\,R_{\mathrm{A}y} & = & 0
\end{array}$$

Equations of equilibrium in matrix form:

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} R_{\mathrm{A}y} \\ R_{\mathrm{A}z} \\ R_{\mathrm{B}x} \\ R_{\mathrm{B}y} \\ R_{\mathrm{B}z} \\ S \end{bmatrix} = \begin{bmatrix} -T_1 \\ 3\,N + 3.5\,T_2 \\ 3.5\,T_1 - N \\ 3\,T_1 + T_2 \\ -N - 3.5\,T_1 \\ 3\,T_1 - T_2 \end{bmatrix}$$

*Notice:* See Matlab file `S3422.m` for numerical solution.

**Solution of Exercise 3.6.2**

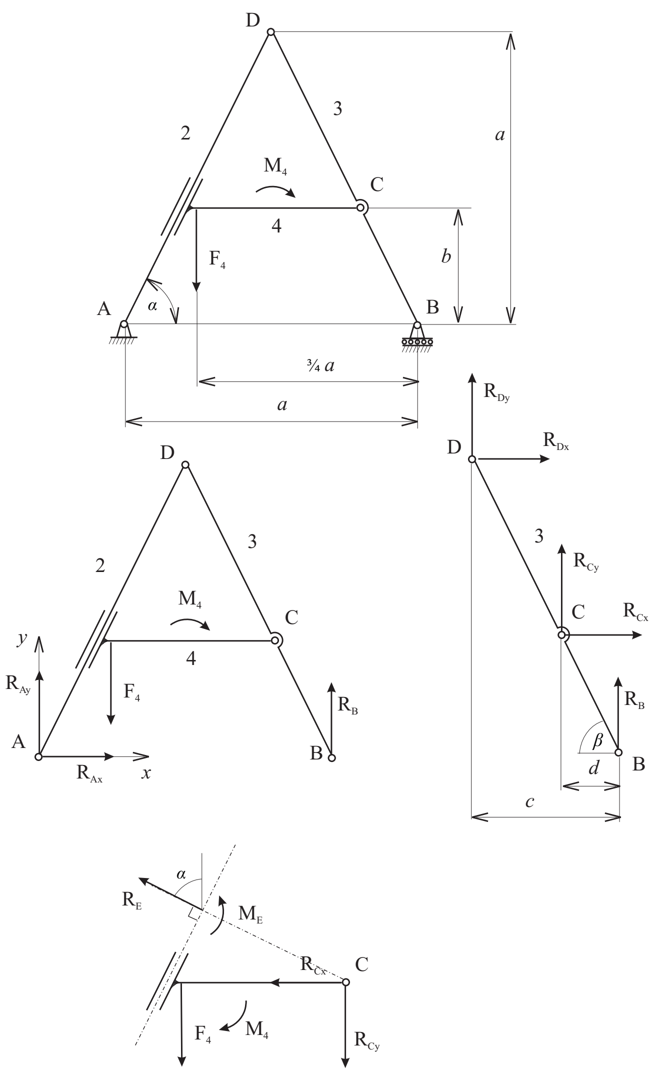


Figure 3.100: Exercise 3.6.2. Equilibrium of a structure

The structure has zero degree of freedom and it consists of three members. We sketch the free-body diagram for the structure as a whole, and for members 3, 4. Than, we write 3 equilibrium equations for each free structure or body. We have

Structure as a whole:

$$\begin{array}{lrcl} \sum F_{ix}: & R_{\mathrm{A}x} & = & 0 \\ \sum F_{iy}: & R_{\mathrm{A}y} + R_{\mathrm{B}} - F_4 & = & 0 \\ \sum M_{i\mathrm{B}}: & R_{\mathrm{A}y} \cdot a - F_4 \cdot 3/4\, a + M_4 & = & 0 \end{array}$$

Member 3:

$$\begin{array}{lrcl} \sum F_{ix}: & R_{\mathrm{D}x} + R_{\mathrm{C}x} & = & 0 \\ \sum F_{iy}: & R_{\mathrm{D}y} + R_{\mathrm{C}y} + R_{\mathrm{B}} & = & 0 \\ \sum M_{i\mathrm{C}}: & R_{\mathrm{B}}\, d - R_{\mathrm{D}y}\,(c-d) - R_{\mathrm{D}x}\,(a-b) & = & 0 \end{array}$$

Member 4:

$$\begin{array}{lrcl} \sum F_{ix}: & -R_{\mathrm{C}x} - R_{\mathrm{E}}\, \sin\alpha & = & 0 \\ \sum F_{iy}: & R_{\mathrm{E}}\, \cos\alpha - R_{\mathrm{C}y} - F_4 & = & 0 \\ \sum M_{i\mathrm{C}}: & M_{\mathrm{E}} + F_4\,(3/4\, a - d) - M_4 & = & 0 \end{array}$$

Geometry yields

$$c = a - \frac{a}{\mathrm{tg}\alpha}\,, \qquad d = \frac{b}{\mathrm{tg}\beta}\,, \qquad \beta = \mathrm{arctg}\frac{a}{c}$$

Equation of equlibrium in matrix form:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & d & 0 & 0 & b-a & d-c & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \sin\alpha & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & \cos\alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R_{\mathrm{A}y} \\ R_{\mathrm{B}} \\ R_{\mathrm{C}x} \\ R_{\mathrm{C}y} \\ R_{\mathrm{D}x} \\ R_{\mathrm{D}y} \\ R_{\mathrm{E}} \\ M_{\mathrm{E}} \end{bmatrix} = \begin{bmatrix} F_4 \\ \frac{3}{4}\, a\, F_4 - M_4 \\ 0 \\ 0 \\ 0 \\ 0 \\ F_4 \\ F_4\,(d - \frac{3}{4}a) + M_4 \end{bmatrix}$$

*Notice:* See Matlab file `SSB612.m` for numerical solution.

**Solution of Exercise 3.6.3**

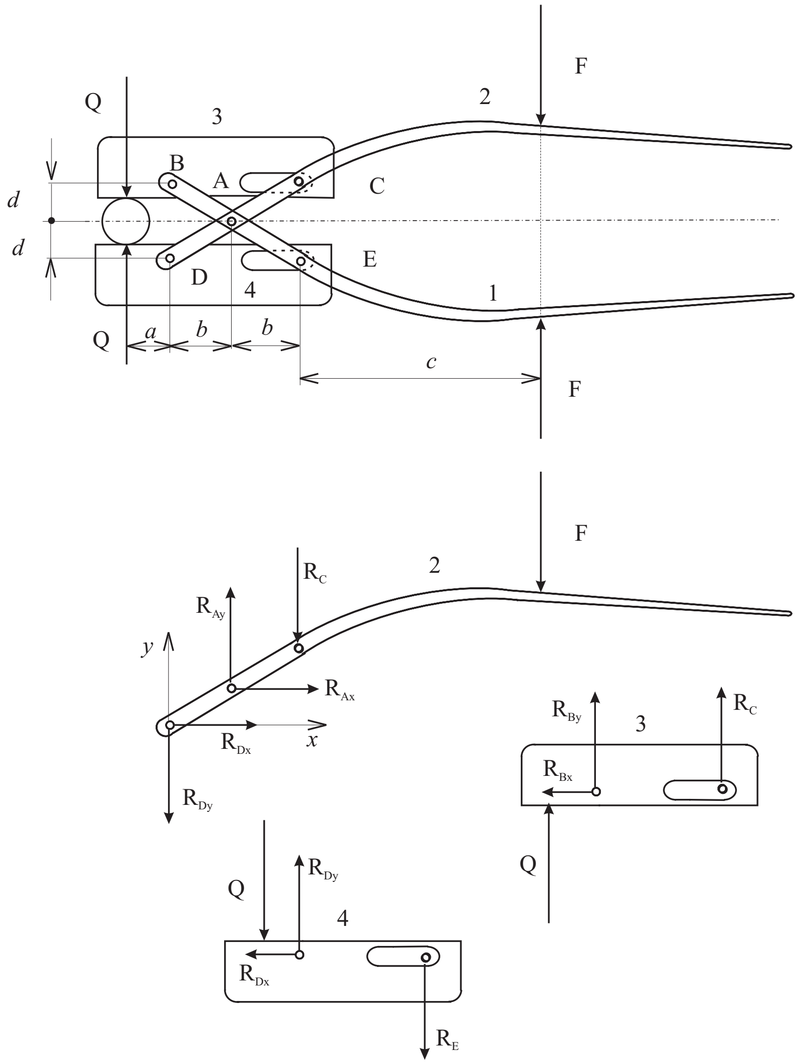


Figure 3.101: Exercise 3.6.3. Equilibrium of the pliers

We free bodies 2, 3, 4 and write the particular equations of equilibrium.
Member 2:

$$
\begin{aligned}
\sum F_{ix}: &\quad R_{\mathrm{A}x} + R_{\mathrm{D}x} &= 0 \\
\sum F_{iy}: &\quad -R_{\mathrm{D}y} + R_{\mathrm{A}y} - R_{\mathrm{C}} - F &= 0 \\
\sum M_{i\mathrm{A}}: &\quad R_{\mathrm{D}x}\, d + R_{\mathrm{D}y}\, b - R_{\mathrm{C}}\, b - F\,(b+c) &= 0
\end{aligned}
$$

Member 3:

$$\begin{aligned}
\textstyle\sum F_{ix}: &\quad R_{\mathrm{B}x} &= 0\\
\textstyle\sum F_{iy}: &\quad Q + R_{\mathrm{B}y} + R_{\mathrm{C}} &= 0\\
\textstyle\sum M_{i\mathrm{B}}: &\quad 2\,b\,R_{\mathrm{C}} - Q\,a &= 0
\end{aligned}$$

Member 4:

$$\begin{aligned}
\textstyle\sum F_{ix}: &\quad R_{\mathrm{D}x} &= 0\\
\textstyle\sum F_{iy}: &\quad -Q + R_{\mathrm{D}y} - R_{\mathrm{E}} &= 0\\
\textstyle\sum M_{i\mathrm{D}}: &\quad Q\,a - 2\,b\,R_{\mathrm{E}} &= 0
\end{aligned}$$

*Remark:* You can see that it is not necessary to free the member 4. It is clear that $R_{\mathrm{D}x} = R_{\mathrm{B}x}, R_{\mathrm{D}y} = R_{\mathrm{B}y}, R_{\mathrm{C}} = R_{\mathrm{E}}$ from symmetry.

Using symmetry and excluding trivial scalar equations we have a system of only 6 equilibrium equation. These are in matrix form:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0\\ 0 & 1 & 0 & -1 & -1 & -1\\ 0 & 0 & d & b & -b & -(b+c)\\ 0 & 0 & 0 & -1 & 1 & 0\\ 0 & 0 & 0 & 0 & 2\,b & 0\\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} R_{\mathrm{A}x}\\ R_{\mathrm{A}y}\\ R_{\mathrm{D}x}\\ R_{\mathrm{D}y}\\ R_{\mathrm{C}}\\ F \end{bmatrix} = \begin{bmatrix} 0\\ 0\\ 0\\ -Q\\ a\,Q\\ Q \end{bmatrix}$$

*Notice:* See Matlab file `SSB614.m` for numerical solution.

**Solution of Exercise 3.6.4**

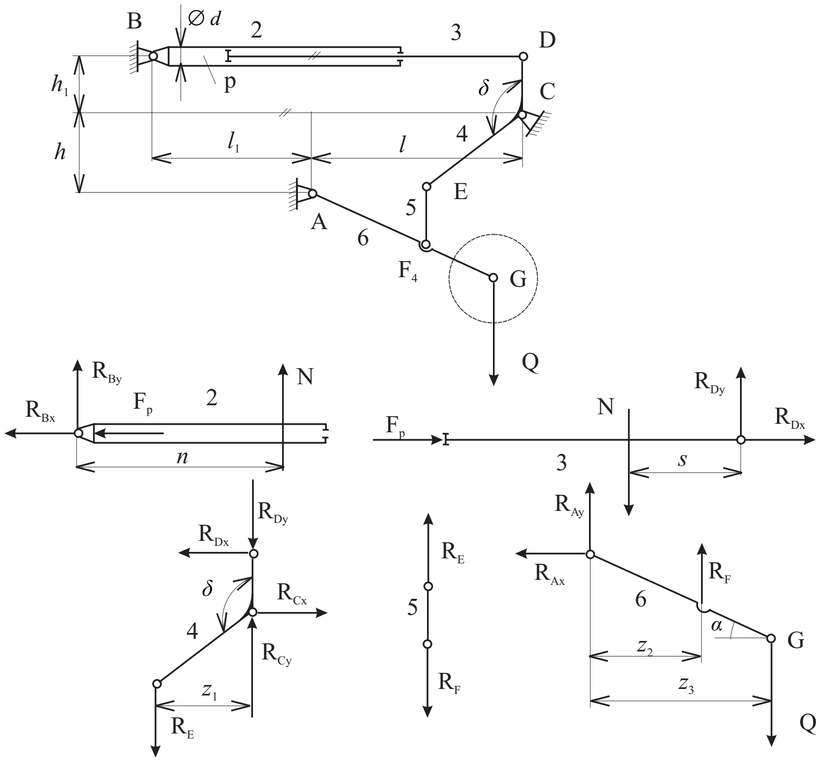


Figure 3.102: Exercise 3.6.4. Equilibrium of the landing gear

First of all we expres the geometrical dependancies.

$$z_1 = \mathrm{EC}\cos\delta\,, \qquad \mathrm{z}_2 = \mathrm{AF}\cos\alpha \qquad \mathrm{z}_3 = \mathrm{AG}\cos\alpha$$

Next we free bodies 2, 3, 4, 5, 6 and write the particular equations of equilibrium.

Member 2:

$$\begin{aligned} \sum F_{ix} &: & -R_{\mathrm{B}x} - F_p &= 0 \\ \sum F_{iy} &: & R_{\mathrm{B}y} + N &= 0 \\ \sum M_{i\mathrm{B}} &: & N\,n &= 0 \end{aligned}$$

Member 3:

$$\begin{aligned} \sum F_{ix} &: & F_p + R_{\mathrm{D}x} &= 0 \\ \sum F_{iy} &: & -N + R_{\mathrm{D}y} &= 0 \\ \sum M_{i\mathrm{D}} &: & N\,n &= 0 \end{aligned}$$

Member 4:

$$\begin{aligned} \sum F_{ix}: \quad & -R_{\mathrm{D}x} + R_{\mathrm{C}x} &= 0 \\ \sum F_{iy}: \quad & -R_{\mathrm{E}} - R_{\mathrm{D}y} + R_{\mathrm{C}y} &= 0 \\ \sum M_{i\mathrm{C}}: \quad & R_{\mathrm{D}x}\, h_1 + R_{\mathrm{E}}\, z_1 &= 0 \end{aligned}$$

Member 5:

$$\sum F_{iy}: \quad R_{\mathrm{E}} - R_{\mathrm{F}} = 0$$

Member 6:

$$\begin{aligned} \sum F_{ix}: \quad & -R_{\mathrm{A}x} &= 0 \\ \sum F_{iy}: \quad & R_{\mathrm{A}y} + R_{\mathrm{F}} - Q &= 0 \\ \sum M_{i\mathrm{A}}: \quad & R_{\mathrm{F}} - z_2 - Q\, z_3 &= 0 \end{aligned}$$

*Remark:* You can see that it is not necessary to determinate angle $\alpha$ . The last equations can be rewriten to

$$R_{\mathrm{F}} = Q \frac{}{AF} BF$$

.

Excluding trivial scalar equations we have system of only 8 equilibrium equation. These are in matrix form:

$$\begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_1 & z_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R_{\mathrm{A}z} \\ R_{\mathrm{B}x} \\ R_{\mathrm{C}x} \\ R_{\mathrm{C}y} \\ R_{\mathrm{D}x} \\ R_E \\ R_F \\ R_{\mathrm{F}p} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ Q \\ Qz_3/z_2 \end{bmatrix}$$

The preasure can be computed from equation:

$$p = \frac{F_p}{\frac{\pi d^2}{4}}$$

.

*Notice:*See Matlab file S615.m for numerical solution.

**Solution of Exercise 3.6.5**

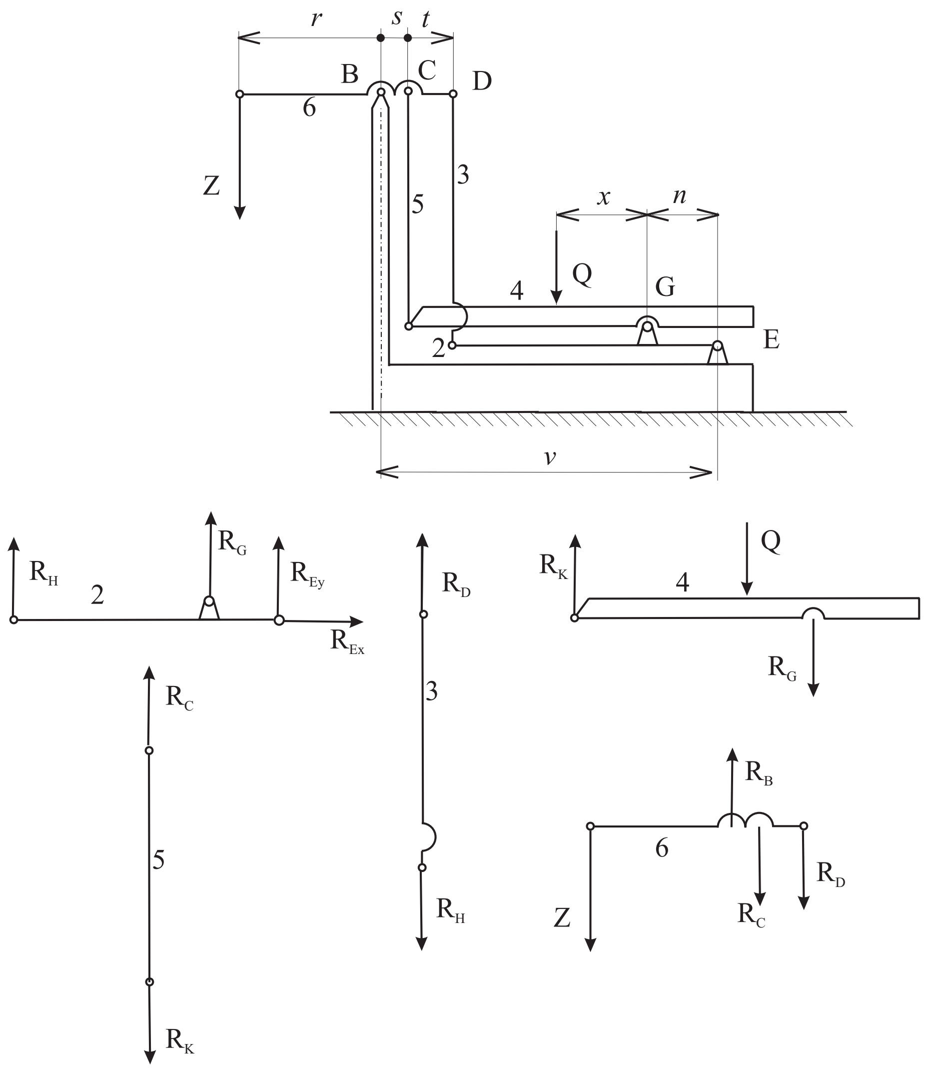


Figure 3.103: Exercise 3.6.5. Equilibrium of the decimal scales

We free bodies 2, 3, 4, 5, 6 and write the particular equations of equilibrium.
Member 2:

$$\begin{aligned}
\sum F_{ix}: & \quad & R_{\text{E}x} &= 0 \\
\sum Fiy: & \quad & R_{\text{G}} + R_{\text{E}y} + R_{\text{H}} &= 0 \\
\sum M_{i\text{H}}: & \quad & R_{\text{G}}\,(v-s-t-u) + R_{\text{E}y}\,(v-s-t) &= 0
\end{aligned}$$

Member 3:

$$\sum F_{iy}: \quad R_{\text{D}} - R_{\text{H}} = 0$$

Member 4:

$$\begin{aligned} \sum F_{iy} : & \quad -R_{\mathrm{G}} + R_{\mathrm{K}} - Q &= 0 \\ \sum M_{i\mathrm{K}} : & \quad -R_{\mathrm{G}y}\,(v-s-u) - Q\,(v-s-u-x) &= 0 \end{aligned}$$

Member 5:

$$\sum F_{iy} : \quad R_{\mathrm{C}} - R_{\mathrm{K}} = 0$$

Member 6:

$$\begin{aligned} \sum F_{iy} : & \quad -Z + R_{\mathrm{B}} - R_{\mathrm{C}} - R_{\mathrm{D}} &= 0 \\ \sum M_{i\mathrm{B}} : & \quad Z\,r - R_{\mathrm{C}}\,s - R_{\mathrm{D}}\,(s+t) &= 0 \end{aligned}$$

Excluding trivial scalar equations we have system of only 8 equilibrium equation. These are in matrix form:

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & v-s-t & v-s-t-u & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & v-s-u & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -s & -(s+t) & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} R_{\mathrm{B}} \\ R_{\mathrm{C}} \\ R_{\mathrm{D}} \\ R_{\mathrm{E}y} \\ R_{\mathrm{G}} \\ R_H \\ R_K \\ Z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ Q \\ Q\,(v-s-u-x) \\ 0 \\ Z \\ Z\,r \end{bmatrix}$$

*Notice:* See Matlab file s6111.m for numerical solution.

.

**Solution of Exercise 3.6.6**

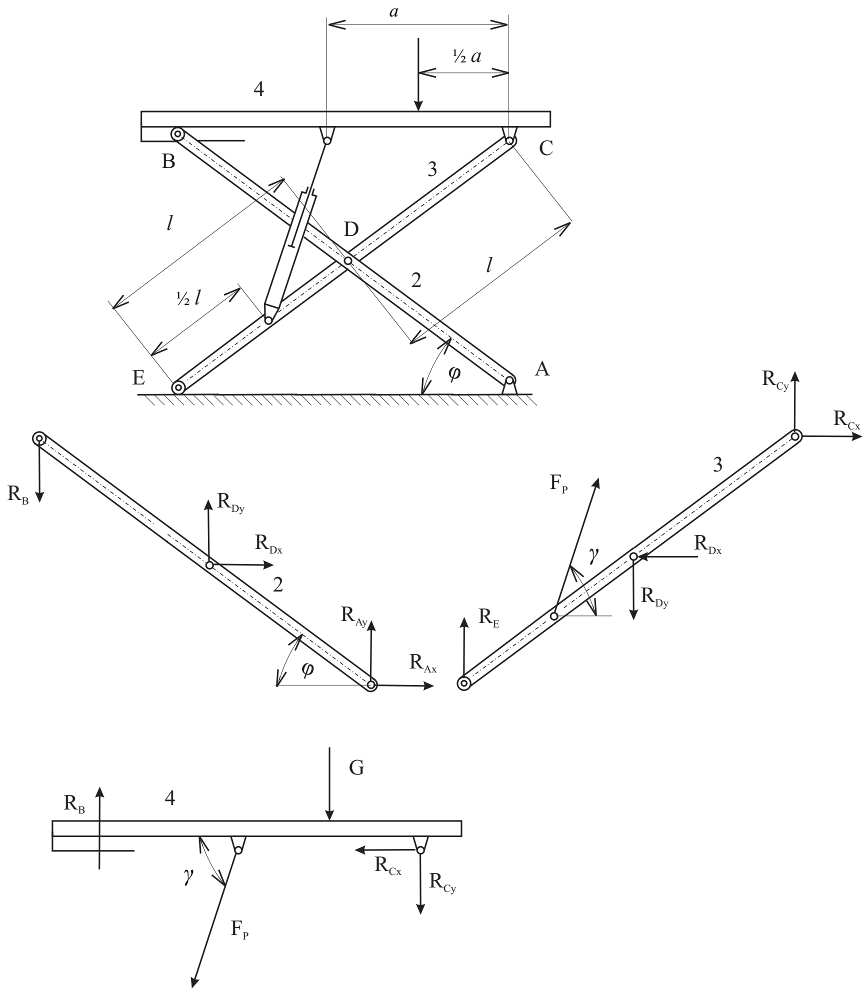


Figure 3.104: Exercise 3.6.6. Equilibrium of the lifting platform

First we express the geometrical dependancies:

$$\eta = \sqrt{(l + \frac{l}{2})^2 + a^2 - 2a(l + \frac{l}{2})cos\phi}$$

$$\gamma = acos(\frac{(l + \frac{l}{2}) * cos\phi - a}{\eta})$$

$$\epsilon = \gamma - \psi$$

Next we free bodies 2, 3, 4, and write the particular equations of equilibrium.
Member 2:

$$\begin{aligned}
\sum F_{ix}: \quad & R_{\mathrm{A}x} + R_{\mathrm{D}x} &= 0\\
\sum F_{iy}: \quad & R_{\mathrm{A}y} + R_{\mathrm{D}y} - R_{\mathrm{B}} &= 0\\
\sum M_{i\mathrm{A}}: \quad & R_{\mathrm{B}}\, 2l\cos\phi - R_{\mathrm{D}y}\, l\cos\phi - R_{\mathrm{D}x}\ l\sin\phi &= 0
\end{aligned}$$

Member 3:

$$\begin{aligned}
\sum F_{ix}: \quad & R_{\mathrm{C}x} - R_{\mathrm{D}x} + F_p\cos\gamma &= 0\\
\sum F_{iy}: \quad & R_{\mathrm{C}y} - R_{\mathrm{D}y} + R_{\mathrm{E}} + F_p\sin\gamma &= 0\\
\sum M_{i\mathrm{C}}: \quad & R_{\mathrm{D}y}\, l\cos\phi - R_{\mathrm{D}x}\, l\sin\phi - R_{\mathrm{E}}\ 2l\cos\phi - F_p\sin(\gamma-\phi)\,\tfrac{l}{2} &= 0
\end{aligned}$$

Member 4:

$$\begin{aligned}
\sum F_{ix}: \quad & -R_{\mathrm{C}x} - F_p\cos\gamma &= 0\\
\sum F_{iy}: \quad & -R_{\mathrm{C}y} - R_{\mathrm{B}} - F_p\sin\gamma &= 0\\
\sum M_{i\mathrm{C}}: \quad & -R_{\mathrm{B}}\, 2lcos\phi + F_p\sin\gamma\ a &= 0
\end{aligned}$$

We have system of only 9 equilibrium equation.
If we introduce auxiliary variables:

$$cp = \cos\phi,\ sp = \sin\phi,\ cg = \cos\gamma,\ sg = \sin\gamma,\ se = \sin\epsilon$$

Can be equations of equlibrium writen in matrix form:

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 2lcp & 0 & 0 & -lsp & -lcp & 0 & 0\\
0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & cg\\
0 & 0 & 0 & 0 & 1 & 0 & -1 & 1 & sg\\
0 & 0 & 0 & 0 & 0 & -lsp & lcp & -2lcp & -se\,(l+\frac{l}{2})\\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -cg\\
0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & -sg\\
0 & 0 & -2lcp & 0 & 0 & 0 & 0 & 0 & sg\,a
\end{bmatrix}
\begin{bmatrix} R_{\mathrm{A}x}\\ R_{\mathrm{A}y}\\ R_{\mathrm{B}}\\ R_{\mathrm{C}x}\\ R_{\mathrm{C}y}\\ R_{\mathrm{D}x}\\ R_{\mathrm{D}y}\\ R_E\\ F_p \end{bmatrix}
=
\begin{bmatrix} 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ 0\\ G\\ -Q\frac{a}{2} \end{bmatrix}$$

*Notice:* See Matlab file S6120.m for numerical solution.

**Solution of Exercise 3.6.7**

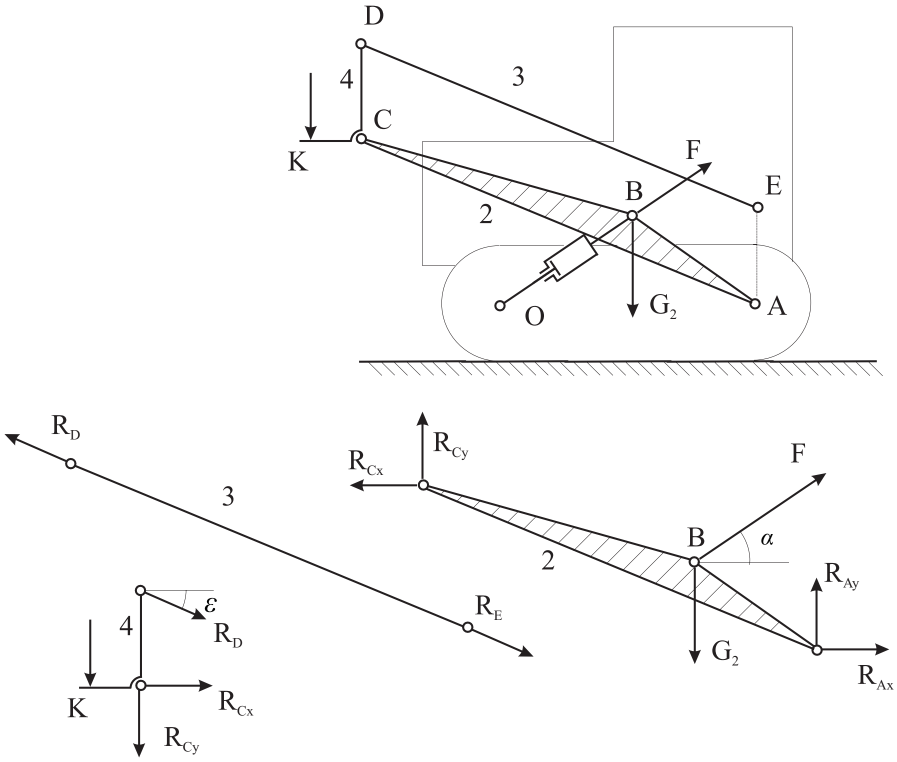


Figure 3.105: Exercise 3.6.7. Equilibrium of a hub lifting mechanism

First we express the geometrical dependancies:

$$\alpha = \mathrm{acos}(\frac{(\mathrm{OB}^2 + \mathrm{OA}^2 - \mathrm{AB}^2)}{2\,\mathrm{OB}\,\mathrm{OA}})$$
$$\delta = \mathrm{acos}(\frac{(\mathrm{AC}^2 + \mathrm{AB}^2 - \mathrm{BC}^2)}{2\,\mathrm{AC}\,\mathrm{AB}})$$
$$\beta = \mathrm{asin}(\frac{\mathrm{OB}}{\mathrm{AB}\,\mathrm{sin}\alpha})$$
$$\epsilon = \beta - \delta$$

Next we free bodies 2, 4, and write the particular equations of equilibrium.
Member 2:

$$\begin{aligned}
\sum F_{ix}: &\quad & R_{\mathrm{A}x} - R_{\mathrm{C}x} + F\,\mathrm{cos}\alpha &= 0\\
\sum F_{iy}: &\quad & R_{\mathrm{A}y} + R_{\mathrm{C}y} + F\,\mathrm{cos}\alpha - \mathrm{G}_2 &= 0\\
\sum M_{i\mathrm{A}}: &\quad & R_{\mathrm{C}x}\,\mathrm{AC}\,\mathrm{cos}\epsilon - \mathrm{R_{Cy}}\,\mathrm{AC}\,\mathrm{sin}\epsilon - \mathrm{F}\,\mathrm{cos}\alpha\,\mathrm{AB}\,\mathrm{sin}\beta - \mathrm{F}\,\mathrm{sin}\alpha\,\mathrm{AB}\,\mathrm{cos}\beta &= 0
\end{aligned}$$

Member 4:

$$\begin{array}{lrcl} \sum F_{ix}: & R_{Cx} + R_{D}\cos\epsilon & = & 0 \\ \sum F_{iy}: & -R_{Cy} - R_{D}\sin\epsilon - Z_4 & = & 0 \\ \sum M_{iC}: & Z_4\,\mathrm{CK} - \mathrm{R_D}\cos\epsilon\,\mathrm{CD} & = & 0 \end{array}$$

We have system of only 6 equilibrium equation.
If we introduce auxiliary variables:

$$mf = -cos\alpha\mathrm{AB}\sin\beta\ - \sin\alpha\mathrm{AB}\cos\beta$$

Can be equations of equlibrium writen in matrix form:

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & \cos\alpha \\ 0 & 1 & 0 & 1 & 0 & \sin\alpha \\ 0 & 0 & \mathrm{AC}\sin\epsilon & -\mathrm{AC}\cos\epsilon & 0 & mf \\ 0 & 0 & 1 & 0 & \cos\epsilon & 0 \\ 0 & 0 & 0 & -1 & -\sin\epsilon & 0 \\ 0 & 0 & 0 & 0 & -\cos\epsilon\,\mathrm{CD} & 0 \end{bmatrix} \begin{bmatrix} R_{Ax} \\ R_{Ay} \\ R_{Cx} \\ R_{Cy} \\ R_{D} \\ F \end{bmatrix} = \begin{bmatrix} 0 \\ \mathrm{G_2} \\ -\mathrm{G_2}\ \mathrm{AB}\cos\beta \\ 0 \\ Z_4 \\ -Z_4\,\mathrm{CK} \end{bmatrix}$$

*Notice:* See Matlab file s6122.m for numerical solution.

**Solution of Exercise 3.8.2**

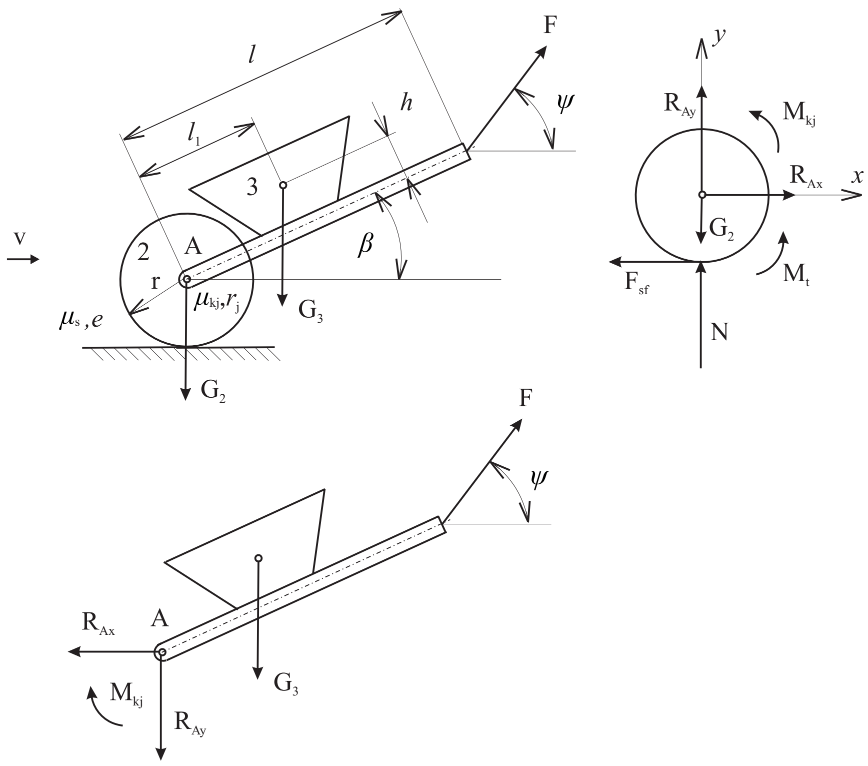


Figure 3.106: Exercise 3.8.2. Equilibrium of a hand-barrow

First we free particular bodies and write down the equations of equilibrium.
Body 2:

$$\begin{aligned}
\sum F_{ix}: &\quad R_{Ax} - F_{sf} &= 0\\
\sum F_{iy}: &\quad R_{Ay} - G_2 + N &= 0\\
\sum M_{iA}: &\quad M_{kj} + M_t - F_{sf}\, r &= 0
\end{aligned}$$

Body 3:

$$\begin{aligned}
\sum F_{ix}: &\quad F\ \cos\psi - \mathrm{R_{Ax}} &= 0\\
\sum F_{iy}: &\quad -R_{Ay} - G_3 + F\sin\psi &= 0\\
\sum M_{iA}: &\quad F\sin\psi\, \mathrm{l}\, \cos\beta - \mathrm{F}\ \cos\psi\, \mathrm{l}\, \sin\beta - \mathrm{G_3}\,(\mathrm{l_1}\ \cos\beta - \mathrm{h}\sin\beta) - \mathrm{M_{kj}} &= 0
\end{aligned}$$

Then we express the friction forces using their definitions

$$M_{kj} = r_j\, \mu_{kj} \sqrt{R_{Ax}^2 + R_{Ay}^2} \quad , \qquad M_t = |N|\, e$$

and we subsitute them into the equations of equilibrium. We get a system of 6 nonlinear algebraic equation containing 6 unknowns $R_{\mathrm{A}x}$, $R_{\mathrm{A}y}$, $F_{\mathrm{sf}}$, $N$, $F$, $\psi$. We solve the system using Matlab. The result is $F$= 154.7 N, $\psi = 84.7°$.

After the solution we check the condition of rolling using the formula

$$|F_{\mathrm{sf}}| \leq |N|\,\mu_{\mathrm{s}} \quad .$$

The condition is valid in our case because $14.34 < 144.36$.

In case we have no solver for system of nonlinear algebraic equations we can use the linearized expression

$$M_{\mathrm{kj}} = r_{\mathrm{j}}\,\mu_{\mathrm{kj}}\,\left(0.96\,|R_{Ay}| + 0.4\,|R_{Ax}|\right)$$

for the moment of friction. Supposing

$$|R_A y| > |R_A x|$$

we get a system of 6 linear algebraic equations after linearizing. These can be written and solved using familiar approach.

Solution of linearizing equations:
$F = 149.53$ N, $\psi = 88.88°$

Solution of nonlinearizing equations:
$F = 158.18$ N, $\psi = 82.33°$

**Solution of Exercise 3.8.3**

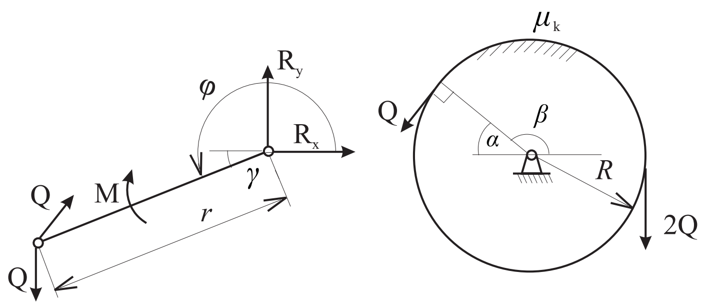


Figure 3.107: Exercise 3.8.3. Free-body diagram

First we use the standard expression for belt friction:

$$\frac{2\,Q}{Q} = e^{\beta\,\mu_\mathrm{k}}$$

This yields

$$\ln 2 = \beta\,\mu_\mathrm{k}$$

$$\beta = \frac{1}{\mu_\mathrm{k}}\ln 2 = 2.310\ \mathrm{rad} = 132.353^\circ$$

From geometry we have

$$\cos\alpha = \frac{R}{r}$$

$$\alpha = \arccos\frac{R}{r} = 60^\circ$$

$$\varphi = \beta + \alpha = 192.353^\circ$$

$$\gamma = \varphi - 180^\circ = 12.353^\circ$$

Equilibrium equation is

$$M + Q\,R - Q\,r\cos\gamma = 0$$

This yields

$$M = Q\,(r\ \cos\gamma - R) = 9.54\ \mathrm{Nm}$$

**Solution of Exercise 3.8.4**

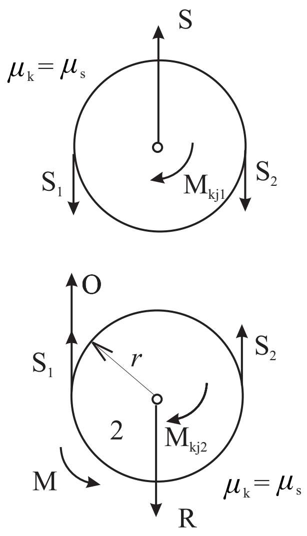


Figure 3.108: Exercise 3.8.4. Free-body diagram

Equations of equilibrium for the upper wheel:

$$\begin{aligned} S &= S_1 + S_2 \\ r_j\,\mu_{kj}\,(S_1 + S_2) &= (S_1 - S_2)\,r \end{aligned}$$

Equations of equilibrium for the lower wheel:

$$\begin{aligned} R &= S_1 + S_2 + O \\ M &= (S_1 + O - S_2)\,r + r_j\,\mu_{kj}\,(S_1 + S_2 + O) \end{aligned}$$

Expression for the belt friction:

$$\frac{S_1 + O}{S_2} = e^{\pi\,\mu_k} \qquad \to \min$$

Altogether we have 5 equations for 5 unknowns, namely $S, S_1, S_2, R, M$. After some manipulations we have

$$\begin{aligned} S_2 &= \frac{r_j\,\mu_{kj} - r}{r_c\,\mu_{kj}\,(e^{\pi\,\mu_k}+1) - r\,(e^{\pi\,\mu_k}-1)]}\,O = 123.1\text{ N} \\ S_1 &= S_2\,e^{\pi\,\mu_k} - O = 130.74\text{N} \\ S &= S_1 + S_2 = 253.846\text{ N} \\ M &= 23.652\text{ Nm} \\ R &= S_1 + S_2 + O = 353.84\text{ N} \end{aligned}$$

**Solution of Exercise 3.8.5**

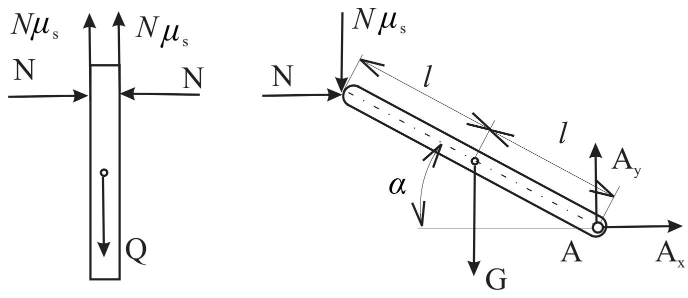


Figure 3.109: Exercise 3.8.5. Free-body diagram

Equation of equilibrium for the plate:

$$Q = 2\,N\,\mu_s$$

Equation of equilibrium for the rod:

$$G\,l\,\cos\alpha + N\,\mu_s\,2\,l\,\cos\alpha = N\,2\,l\,\sin\alpha$$

Solution:

$$N = \frac{G\,\cos\alpha}{2\,(\sin\alpha - \mu_s\,\cos\alpha)} = 116.839\ \mathrm{N}$$
$$Q = 2\,N\,\mu_s = 35.052\ \mathrm{N}$$

The condition for $\alpha_{\max}$ is

$$N \to \infty$$

Hence:

$$\tan\alpha_{\max} = \mu_s$$

and

$$\alpha_{\max} = 8.53^\circ$$

**Solution of Exercise 3.8.6**

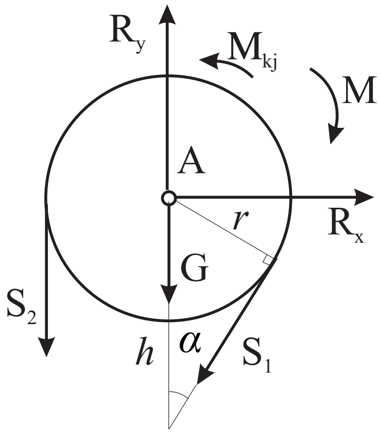


Figure 3.110: Exercise 3.8.6. Free-body diagram

Equations of equilibrium :

$$\begin{aligned} R_x - S_1\,\sin\alpha &= 0 \\ R_y - G - S_2 - S_1\,\cos\alpha &= 0 \\ -M + M_{\mathrm{kj}} + (S_2 - S_1)\,r &= 0 \end{aligned}$$

The expressions for friction forces are:

$$M_{\mathrm{kj}} = r_{\mathrm{j}}\,\mu_{\mathrm{kj}}\,R$$

$$\frac{S_2}{S_1} = e^{(\pi+\alpha)\,\mu_{\mathrm{k}}}$$

Using Poncelet expression for linearization of friction moment we write

$$M_{kj} = r_j\,\mu_{\mathrm{kj}}\,(0.96\,(G + S_2 + S_1\,\cos\alpha) + 0.4\,S_1\,\sin\alpha)$$

Geometry yields

$$\sin\alpha = \frac{r}{h} = \frac{1}{2} => \alpha = \frac{\pi}{6}$$

hence

$$S_2 = S_1\,e^{\frac{7\,\pi}{6}\,0.3} = 3.003\,S_1$$

After substitution we have

$$M_{kj} = r_f\,\mu_{\mathrm{kj}}\,(0.96\,(G + S_1\,(e^{\frac{7\,\pi}{6}\,0.3} + \cos\alpha)) + 0.4\,S_1\,\sin\alpha)$$

and

$$M_{kj} + S_1\,(e^{\frac{7\,\pi}{6}\,0.3} - 1)\,r = M$$

$$S_1\,[0.96\,r_j\,\mu_{kj}\,(e^{\frac{7\,\pi}{6}\,0.3}+\cos\alpha))+r_j\,\mu_{kj}\,0.4\,\sin\alpha+(e^{\frac{7\,\pi}{6}\,0.3}-1)\,r = M-0.96 r_j\,\mu_{kj}\,G$$

$$S_1 = \frac{M - 0.96\,r_{\mathrm{j}}\,\mu_{\mathrm{kj}}\,G}{0.96\,r_{\mathrm{j}}\,\mu_{\mathrm{kj}}\,(e^{\frac{7\,\pi}{6}\,0.3} + \cos\alpha)) + r_j\,\mu_{kj}\,0.4\,\sin\alpha + (e^{\frac{7\,\pi}{6}\,0.3} - 1)\,r} = 792.385\,\mathrm{N}$$

At the end

$$P\,l = S_1\,r\,\cos 30^\circ$$

$$P = \frac{r}{l}\,S_1\,\cos 30^\circ = 214.44\ \mathrm{N}$$

**Solution of Exercise 3.8.7**

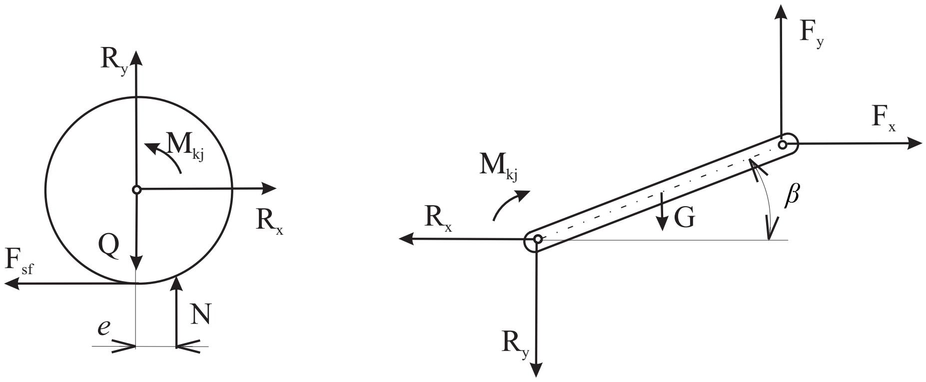


Figure 3.111: Exercise 3.8.7. Free-body diagram

Equations of equilibrium of the roller:

$$\begin{aligned} R_x - F_{\mathrm{sf}} &= 0 \\ R_y + N - Q &= 0 \\ M_{\mathrm{kj}} + N\,e - F_{\mathrm{sf}}\,r &= 0 \end{aligned}$$

Equations of equilibrium of the tow bar:

$$\begin{aligned} F_x - R_x &= 0 \\ F_y - G - R_y &= 0 \\ -M_{\mathrm{kj}} + F_y\,2\,l\,\cos\beta - F_x\,2\,l\,\sin\beta + G\,l\,\cos\beta &= 0 \end{aligned}$$

Poncelet expression for the friction moment:

$$M_{\mathrm{kj}} = r_{\mathrm{j}}\,\mu_{\mathrm{kj}}\,[0.96\,R_y + 0.4\,R_x]$$

Equations of equilibrium in matrix form:

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ -r & e & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & -2l\sin\beta & 2l\cos\beta & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -r_{\mathrm{j}}\mu_{\mathrm{kj}}0.4 & -r_{\mathrm{j}}\mu_{\mathrm{kj}}0.96 & 1 \end{bmatrix} \begin{bmatrix} T_{sf} \\ N \\ F_x \\ F_y \\ R_x \\ R_y \\ M_{\mathrm{kj}} \end{bmatrix} = \begin{bmatrix} 0 \\ Q \\ 0 \\ 0 \\ G \\ -G\,l\,\cos\beta \\ 0 \end{bmatrix}$$

*Notice:* See Matlab file S736.m for numerical solution.

**Solution of Exercise 3.9.2**

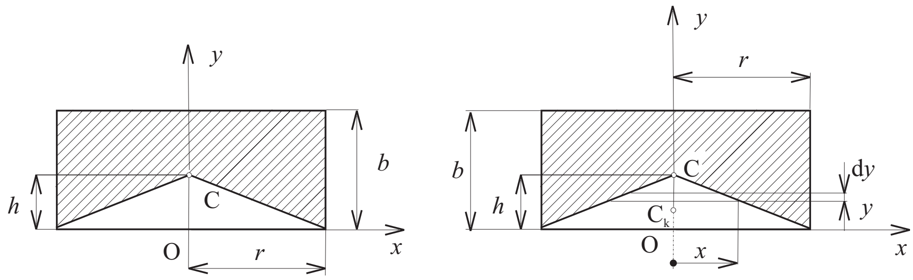


Figure 3.112: Exercise 3.9.2. The centroid of a flywheel

The flywheel is composed from a cylinder from which a cone is extracted. The centroid is located on the axis $y$ due to symmetry and the following is valid

$$y_{\mathrm{C}} = \frac{y_{\mathrm{C1}}\, V_1 - y_{\mathrm{C2}}\, V_2}{V_1 - V_2} = h \tag{3.54}$$

where the subscript 1 denotes the cylinder and the subscript 2 denotes the cone. According to Fig. 3.112 we have

$$y_{\mathrm{C2}} = \frac{\int\limits_{V_2} y\, dV}{V_2} = \frac{\int\limits_0^h y\pi \frac{r^2}{h^2}(h-y)^2 dy}{\frac{1}{3}\,\pi r^2 h} = \frac{1}{4}\, h \tag{3.55}$$

The substitution 3.55 to 3.54 yields

$$h = \frac{\frac{b}{2}\,\pi r^2 b - \frac{h}{4}\cdot\frac{1}{3}\,\pi r^2 h}{\pi r^2 b - \frac{1}{3}\,\pi r^2 h}$$

and after some manipulation we have

$$h^2 - 4bh + 2b^2 = 0$$

The root $h = b\,(2-\sqrt{2}) = 0.586\,b$ is acceptable.

*Notice:* See Matlab file `SCG102.m` for numerical solution.

**Solution of Exercise 3.9.3**

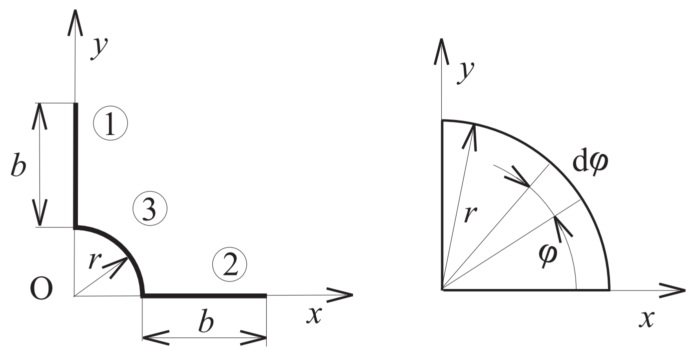


Figure 3.113: Exercise 3.9.3. Division of the wire

We split the wire into three parts:

$$\begin{array}{lll} \mathrm{Part1}: & x_{C1} = 0, & y_{C1} = r + \frac{b}{2} \\ \mathrm{Part2}: & x_{C2} = r + \frac{b}{2}, & y_{C2} = 0 \\ \mathrm{Part3}: & x_{C3} = \frac{2\,r}{\pi}, & y_{C3} = \frac{2\,r}{\pi} \end{array}$$

To compute $x_{C3}$ we can write

$$x_{C3}\,\frac{\pi}{2}\,r = \int_0^{\frac{\pi}{2}} r\,\cos\,\varphi\,r\,\mathrm{d}\varphi = r^2\,[\sin\varphi]_0^{\frac{\pi}{2}} = r^2$$

and hence

$$x_{C3} = \frac{2r}{\pi}$$

Due to symmetry

$$y_{C3} = x_{C3}$$

.

To compute $x_C$ we write

$$\begin{aligned} x_C\,l &= x_{C1}\,l_1 + x_{C2}\,l_2 + x_{C3}\,l_3 \\ x_C(2\,b + \frac{\pi\,r}{2}) &= 0 + (r + \frac{b}{2})\,b + \frac{2\,r}{\pi}\,\frac{\pi\,r}{2} \\ x_C &= 0.0334\ \mathrm{m} \end{aligned}$$

Due to symmetry

$$y_C = x_C$$

.

**Solution of Exercise 3.9.4**

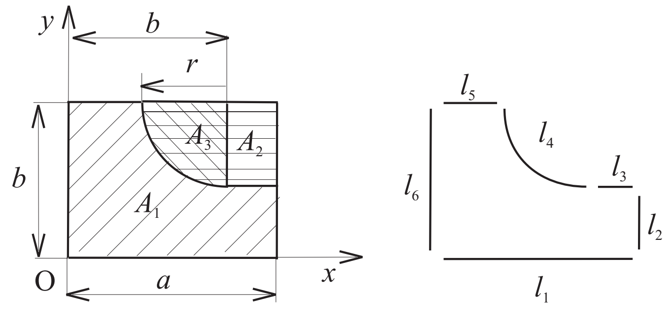


Figure 3.114: Exercise 3.9.4. Division of the area

We solve the excercise for the centre of gravity of the area first.
We split the area into three parts. Areas of the particular parts are:

$$A_1 = a\,b; \qquad A_2 = (a-b)\,r; \qquad A_3 = \frac{\pi\,r^2}{4}$$

and the interesting area $A$ is

$$A = A_1 - A_2 - A_3$$

Coordinates of centers of gravity are

$$\begin{array}{lll} \text{Part 1}: & x_{C1} = \frac{a}{2}, & y_{C1} = \frac{b}{2} \\ \text{Part 2}: & x_{C2} = \frac{a+b}{2}, & y_{C2} = b - \frac{r}{2} \\ \text{Part 3}: & x_{C3} = b - \frac{4\,r}{3\,\pi}, & y_{C3} = b - \frac{4\,r}{3\,\pi} \end{array}$$

To compute $x_C$ we write

$$x_{C_{area}}\,A = x_{C1}\,A_1 - x_{C2}\,A_2 - x_{C3}\,A_3$$

To compute $y_C$ we write

$$y_{C_{area}}\,A = y_{C1}\,A_1 - y_{C2}\,A_2 - y_{C3}\,A_3$$

After substitution of numerical values we have the result

$$x_{C_{area}} = 0.02714\ \mathrm{m} \qquad y_{C_{area}} = 0.01343\ \mathrm{m}$$

Second we solve the problem of centres of gravity of circumference. We split the area into six parts. Data necessary for computation are in table:

| Part No. | $l_i$ | $x_{\mathrm{C}i}$ | $y_{\mathrm{C}i}$ |
|---|---|---|---|
| 1 | $a$ | $\frac{a}{2}$ | 0 |
| 2 | $b-r$ | $a$ | $\frac{b-r}{2}$ |
| 3 | $a-b$ | $\frac{a+b}{2}$ | $b-r$ |
| 4 | $\frac{\pi\,r}{2}$ | $b-\frac{2\,r}{\pi}$ | $b-\frac{2\,r}{\pi}$ |
| 5 | $b-r$ | $\frac{b-r}{2}$ | $b$ |
| 6 | $b$ | 0 | $\frac{b}{2}$ |

To compute $x_{\mathrm{C_{circu}}}$ and $y_{\mathrm{C_{circu}}}$ we write

$$x_{\mathrm{C_{circu}}}\,l = \sum_{1}^{6} x_{\mathrm{C}i}\,l_i; \qquad y_{\mathrm{C_{circu}}}\,l = \sum_{1}^{6} y_{\mathrm{C}i}\,l_i$$

After substitution of numerical values we have the result

$$x_{\mathrm{C_{circu}}} = 0.0296\ \mathrm{m} \qquad y_{\mathrm{C_{circu}}} = 0.0138\ \mathrm{m}$$

**Solution of Exercise 3.9.5**

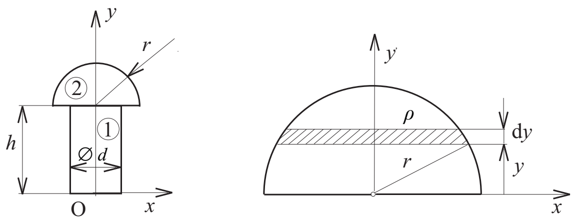


Figure 3.115: Exercise 3.9.5. Decomposition of a rivet

We decompose the rivet into two parts. Part 1 is the cylinder, part 2 is the semisphere.

$$\text{Part 1 :}\quad V_1 = \tfrac{\pi\, d^2}{4}\, h;\quad y_{C1} = \tfrac{h}{2}$$
$$\text{Part 2 :}\quad V_2 = \tfrac{2}{3}\, \pi\, r^3;\quad y_{C2} = \tfrac{3\, r}{8}$$

The centroid of the semisphere we compute as follows:

$$y_{C2} V_2 = \int y \,\mathrm{d}V = \int y\pi\, \rho^2 \mathrm{d}y = \int y\pi\, (r^2 - y^2)\mathrm{d}y = \int_0^r \pi\, r^2\, y\, \mathrm{d}y - \int_0^r \pi\, y^3\, \mathrm{d}y =$$

$$= \frac{1}{2}\pi\, r^4 - \frac{1}{4}\pi\, r^4 = \frac{1}{4}\pi\, r^4$$

Hence

$$y_{C2} = \frac{\frac{1}{4}\, \pi\, r^4}{\frac{2}{3}\, \pi\, r^3} = \frac{3\, r}{8}$$

The coordiante $y_C$ of the centroid of the whole rivet we compute from the equation

$$y_C\, V = y_{C1}\, V_1 + y_{C2}\, V_2$$

After substitutions we have

$$y_C\, (\frac{\pi\, d^2}{4}\, h + \frac{\pi\, d^3}{12}) = \frac{h}{2}\, \frac{\pi\, d^2}{4}\, h + (h + \frac{3\, r}{8})\, \frac{2}{3}\, \pi\, r^3$$

The result is

$$y_C = 0.0476 m$$

**Solution of Exercise 3.9.6**

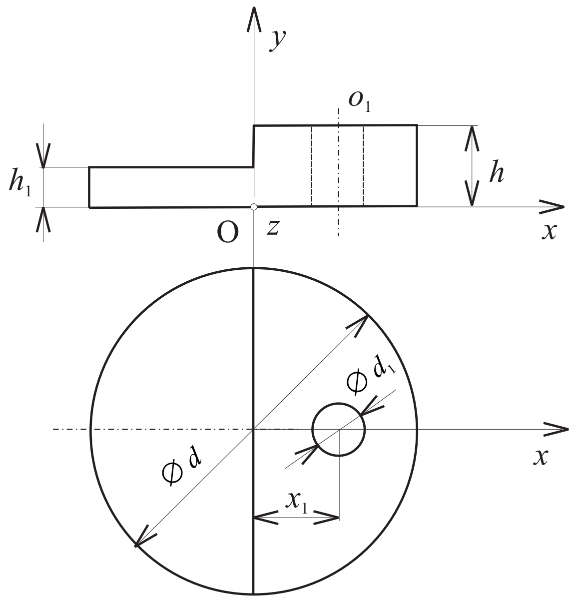


Figure 3.116: Exercise 3.9.6. The particular volumes

We first split the circular plate into three parts:

$$\begin{array}{lll} \text{Part1}: & V_1 = \frac{\pi\, d^2}{4}\, h; & x_{C1} = 0 \\ \text{Part2}: & V_2 = -\frac{\pi\, d^2}{8}\, (h - h_1); & x_{C2} = -\frac{2\, d}{3\, \pi} \\ \text{Part3}: & V_3 = -\frac{\pi\, d_1^2}{4}\, h; & x_{C3} = x_1 \end{array}$$

For $x_C$ we have

$$x_C\, V = x_{C1}\, V_1 + x_{C2}\, V_2 + x_{C3}\, V_3$$

Using the condition

$$x_C = 0$$

we find that

$$\frac{2}{3}\, \frac{d}{\pi}\, \frac{\pi d^2}{8}\, (h - h_1) - x_1\, \frac{\pi\, d_1^2}{4}\, h = 0$$

From the last equation we compute

$$d_1 = \sqrt{\frac{(h - h_1)\, d^3}{3\, \pi\, x_1\, h}} = 0.0583 \text{ m}$$

*Notice:* No Matlab file is necessary.

## Solution of Exercise 3.10.2

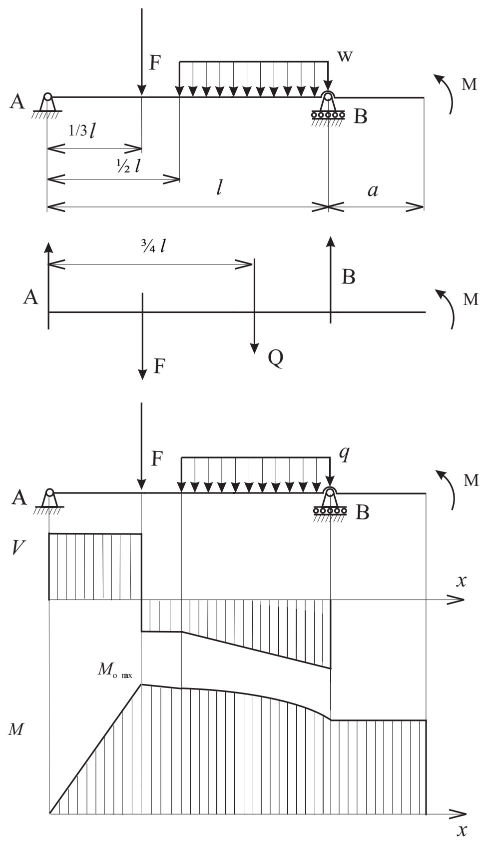


Figure 3.117: Exercise 3.10.2. Internal forces in a beam

**Solution**

We compute reaction forces $R_\mathrm{A}, R_\mathrm{B}$ first. We replace the distributed load by a force $W = w\,\frac{l}{2} = 30$ N the point of action of which is in the centroid of the rectangle (see Fig. 3.68). Using moment equlibrium equation we have

$$\begin{aligned} \sum M_{iA}: &\quad M + R_\mathrm{B}\, l - W\,\tfrac{3}{4}\, l - F\,\tfrac{l}{3} = 0 \\ \sum M_{iB}: &\quad R_\mathrm{A}\, l - F\tfrac{2}{3}\, l - W\,\tfrac{1}{4}\, l - M = 0 \end{aligned}$$

The result is $R_\mathrm{A} = 174.16$ N, $R_\mathrm{B} = 55.83$ N.

We compute the particular internal forces in intervals where no change of load type occurs using definitions.

Interval $0 < x < \frac{l}{3}$:

$$\begin{aligned} N &= 0 \\ V &= R_\mathrm{A} = 174.16\,\mathrm{N} \\ M_\mathrm{b} &= R_\mathrm{A}\,x = (174.16\,x)\,\mathrm{Nm} \end{aligned}$$

Interval $\frac{l}{3} < x < \frac{l}{2}$:

$$\begin{aligned} N &= 0 \\ V &= R_\mathrm{A} - F = -25.840\,\mathrm{N} \\ M_\mathrm{b} &= R_\mathrm{A}\,x - (x - \tfrac{l}{3})\,F = (-25,84\,x + 40)\,\mathrm{Nm} \end{aligned}$$

Interval $\frac{l}{2} < x < l$:

$$\begin{aligned} N &= 0 \\ V &= R_\mathrm{A} - F - w\,(x - \tfrac{l}{2}) = (4,16 - x)\,\mathrm{N} \\ M_\mathrm{b} &= R_\mathrm{A}\,x - (x - \tfrac{l}{3})\,F - w\,(x - \tfrac{l}{2})\tfrac{1}{2}(x - \tfrac{l}{2}) \end{aligned}$$

Interval $l < x < l + a$:

$$\begin{aligned} N &= 0 \\ V &= R_\mathrm{A} - F - w\,\tfrac{l}{2} + R_\mathrm{B} = 0\,\mathrm{N} \\ M_\mathrm{b} &= R_\mathrm{A}\,x - F\,(x - \tfrac{l}{3}) - w\,l\tfrac{1}{2}(x - \tfrac{3}{4}\,l) + R_\mathrm{B}\,(x - l) \end{aligned}$$

The plot of results is shown in Fig.3.68. You can see that maximum bending moment $M_\mathrm{bmax}$ is in position

$$x = \frac{l}{3} = 0.2\ \mathrm{m}$$

where $V = 0$ occurs. Its value is $M_\mathrm{omax} = 34.832$ Nm.

*Notice:* See Matlab file `beam2D.m` for numerical solution.

**Solution of Exercise 3.11.2**

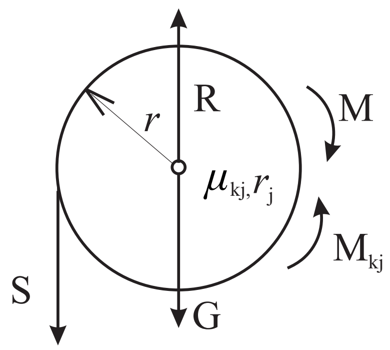


Figure 3.118: Exercise 3.11.2. Free-body diagram

Equation of equilibrium:

$$M = S\,r + M_{\mathrm{kj}}$$

The force in the spring:

$$S = k\,x$$

The moment of friction:

$$M_{kj} = r_{\mathrm{j}}\,\mu_{\mathrm{kj}}\,R = r_{\mathrm{j}}\,\mu_{\mathrm{kj}}\,(S + G)$$

The moment $M$ as a function of $x$:

$$M = k\,x\,r + r_{\mathrm{j}}\,\mu_{\mathrm{jk}}\,(k\,x + G)$$

Mechanical work of $M$:

$$W = \int M\,\mathrm{d}\varphi$$

Geometry:

$$x = r\,\varphi, \quad \mathrm{d}x = r\,\mathrm{d}\varphi$$

Computation:

$$W = \int M\,\frac{dx}{r} = \int_0^h [\frac{r_{\mathrm{j}}\,\mu_{\mathrm{kj}}}{r}\,G + k\,(1 + \frac{r_{\mathrm{j}}\,\mu_{\mathrm{jk}}}{r})\,x]\,\mathrm{d}x$$

$$W = \frac{r_j\,\mu_{\mathrm{jk}}}{r}\,G\,h + \frac{1}{2}\,k\,(1 + \frac{r_{\mathrm{j}}\,\mu_{\mathrm{kj}}}{r})\,h^2$$

Result:

$$W = 15.3\;\mathrm{Nm}$$

**Solution of Exercise 3.11.3**

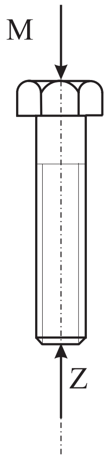


Figure 3.119: Exercise 3.11.3. Free-body diagram

Moment:

$$M = Z\,r\,\tan(\alpha+\varphi)$$

The force in spring:

$$Z = k\,x$$

The friction angle:

$$\varphi = \arctan\mu_{\mathrm{k}} = 2.862^\circ$$

Geometry:

$$\tan\alpha = \frac{x}{r\,\varphi}, \quad \varphi = \frac{x}{r\tan\alpha}, \quad \mathrm{d}\varphi = \frac{1}{r\tan\alpha}\,\mathrm{d}x$$

After substitution we have:

$$M = k\,x\,r\,\tan(\alpha+\varphi)$$

Mechanical work of $M$:

$$W = \int M\,\mathrm{d}\varphi = \int \frac{M}{r\,\tan\alpha}\,\mathrm{d}x$$

$$W = \int_0^h k\,x\,\frac{\tan(\alpha+\varphi)}{\tan\alpha}\,\mathrm{d}x = \frac{1}{2}\,k\,h^2\,\frac{\tan(\alpha+\varphi)}{\tan\alpha}$$

Result:

$$W = 11.6\ \mathrm{Nm}$$

**Solution of Exercise 3.11.4**

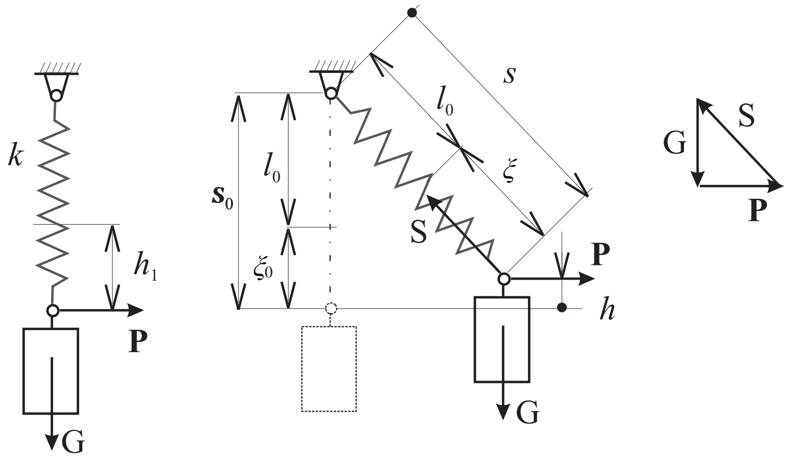


Figure 3.120: Exercise 3.11.4. Free-body diagram

First we redraw the body into the current position and we sketch all relevant forces, namely $G, P, S$. Mechanical work of the force $P$ is the summ of mechanical work $W_1$ needed for lifting the body, and mechanical work $W_2$ needed for stretching of the spring.

Mechanical work $W_1$:

$$W_1 = \int_0^{h_1} G \, \mathrm{d}h = G \, h_1 = 10 \,.\, 0,2 = 2 \text{ Nm}$$

Mechanical work $W_2$:

$$W_2 = \int_{s_0}^{s_1} S \, \mathrm{d}s = \int_{\xi_0}^{\xi_1} k \, \xi \, \mathrm{d}\xi = \frac{1}{2} \, k \, (\xi_1^2 - \xi_0^2)$$

where $\xi, \ \xi_0, \ \xi_1$ denote deformations of the spring in current, original, and end positions.
Geometry:

$$\frac{l_0 + \xi}{l_0 + \xi_0 - h} = \frac{S}{G}$$

As $S = k \, \xi$, $G = k \, \xi_0$ the following is valid

$$\xi = \frac{G \, l_0}{k \, (l_0 - h)}$$

and for $h = h_1$ we have

$$\xi_1 = \frac{G\,l_0}{k\,(l_0 - h_1)}$$

$$W_2 = \frac{1}{2}\,k\,(\xi_1^2 - \xi_0^2) = \frac{G^2}{2\,k}\left[\frac{l_0^2}{(l_0 - h_1)^2} - 1\right] = \frac{10^2}{2\,.\,100}\,\left[\frac{0.3^2}{(0.3 - 0.2)^2} - 1\right] = 4\,\mathrm{Nm}$$

Mechanical work $W_P$ of the force $P$:

$$W_P = W_1 + W_2 = 6\ \mathrm{Nm}$$

**Solution of Exercise 3.11.5**

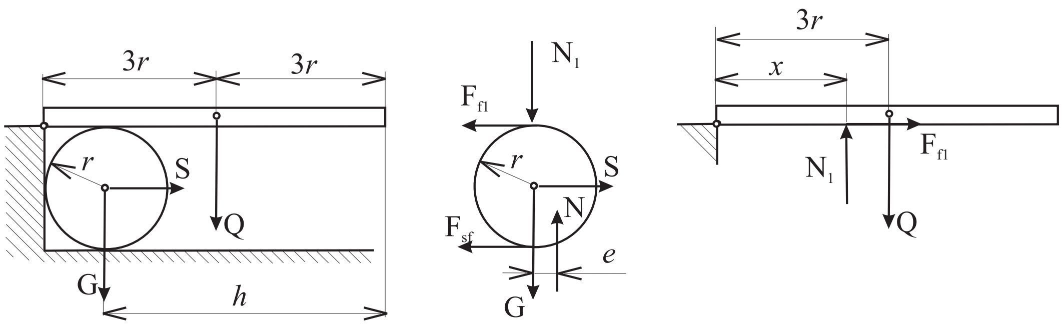


Figure 3.121: Exercise 3.11.5. Free-body diagram

Mechanical work of the force $S$ done along the path $h$ is

$$W_S = \int_0^h S \,\mathrm{d}x$$

First we have to express the magnitude of the force $S$ as a function of $x$. We free the system of bodies in the current position for the purpose. We suppose rolling of the cylinder without slipping on the ground.

Equations of equilibrium of the cylinder:

$$\begin{aligned} S - F_{\mathrm{sf}} - F_{\mathrm{f1}} &= 0 \\ N - G - N_1 &= 0 \\ F_{\mathrm{f1}}\, r + N\, e - F_{\mathrm{sf}}\, r &= 0 \end{aligned}$$

Equation of equilibrium of the plate:

$$N_1\, x - Q\, 3\, r = 0$$

Friction force:

$$F_{\mathrm{f1}} = N_1\, \mu_{\mathrm{k}}$$

After substitution and some manipulations we have:

$$S = Q\, 3\, r \left( 2\, \mu_{\mathrm{k}} + \frac{e}{r} \right) \frac{1}{x} + G\, \frac{e}{r}$$

hence

$$\begin{aligned}
W_S &= \int_r^{6\,r} S\,\mathrm{d}x = Q\,3\,r\,\left(2\,\mu_\mathrm{k} + \tfrac{e}{r}\right)\int_r^{6\,r}\tfrac{\mathrm{d}x}{x} + G\,\tfrac{\xi}{r}\int_r^{6\,r}\mathrm{d}x \\
W_S &= Q\,3\,r\,\left(2\,\mu_\mathrm{k} + \tfrac{\xi}{r}\right)\ln 6 + \mathrm{G}\,\xi\,5 \\
W_S &= 500\,.\,3\,.\,0.1\,\left(2\,.\,0.3 + \tfrac{0.01}{0.1}\right)\ln 6 + 300\,.\,0.01\,.\,5 \\
W_S &= 203.13\ \mathrm{Nm}
\end{aligned}$$

**Solution of Exercise 3.11.6**

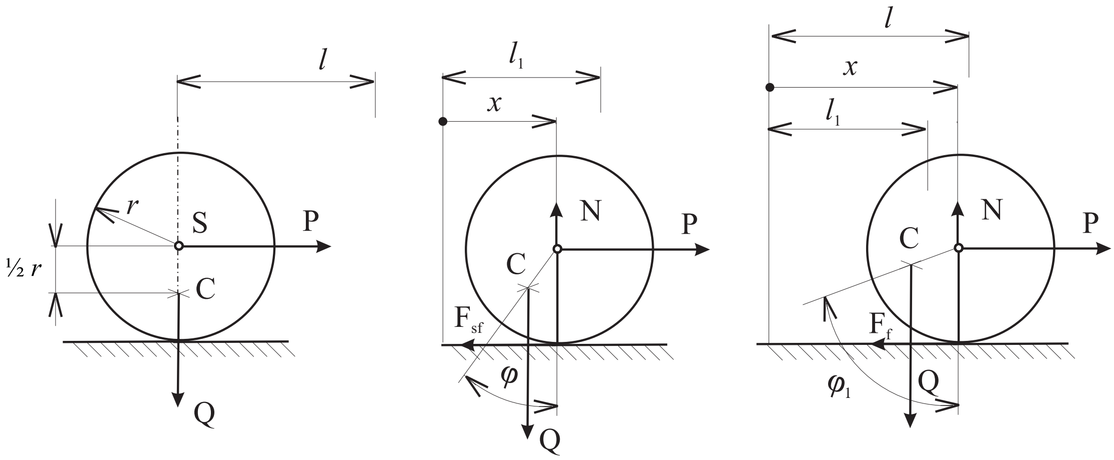


Figure 3.122: Exercise 3.11.6. Free-body diagram

The cylinder will roll without slipping at the beginning of its motion due to the magnitude of $\mu_\mathrm{s}$ given. Hence the equations of equilibrium of the cylinder are

$$P - F_\mathrm{sf} = 0 \,, \qquad N - Q = 0 \,, \qquad Q\,\frac{r}{2}\,\sin\varphi - F_\mathrm{s}\,r = 0$$

and

$$P = \frac{Q}{2}\,\sin\varphi$$

Simultaneously the condition for rolling have to be fulfilled:

$$F_\mathrm{s} \leq N\,\mu_\mathrm{s} \,, \qquad F_\mathrm{s} \leq Q\,\mu_\mathrm{sf}$$

The maximum angle $\varphi_1$ follows from the maximum value of $F_\mathrm{sf}$ which is

$$F_\mathrm{sfmax} = Q\,\mu_\mathrm{sf} = \frac{Q}{2}\,\sin\varphi_1$$

$$2\,\mu_\mathrm{sf} = \sin\varphi_1$$

For $\mu_\mathrm{s} = 0.25$, $\sin\varphi_1 = 0.5$, $\varphi_1 = 30°$. It follows that in the first stage of motion the coordinate $x$ changes from $0$ to $l_1 = r\,\varphi_1$.
Geometry gives

$$x = r\,\varphi, \qquad \mathrm{d}x = r\,\mathrm{d}\varphi$$

Mechanical work of the force $P$ during the first stage of motion is

$$\begin{aligned} W_{P1} &= \int_0^{l_1} P\,\mathrm{d}x = \frac{Q\,r}{2}\int_0^{\varphi_1} \sin\varphi\,\mathrm{d}\varphi \\ W_{P1} &= Q\,\frac{r}{2}\,(1-\cos\varphi_1) = 80\,\frac{0.3}{2}\,(1-\cos 30°) \quad \mathrm{Nm} \\ W_{P1} &= 1.607\ \mathrm{Nm} \end{aligned}$$

During the second stage of motion cylinder slips on the ground along the path $l_1 \to l$ due to force $P = Q\,\mu_k$ magnitude of which is constant.

Mechanical work of the force $P$ during the second stage of motion is

$$\begin{aligned} W_{P2} &= P\int_{l_1}^{l} \mathrm{d}x = Q\,\mu_\mathrm{k}\,(l - r\,\varphi_1) \\ W_{P2} &= 80\,.\,0.25\,\left(1 - 0.3\,\frac{30\,\pi}{180}\right) \quad \mathrm{Nm} \\ W_{P2} &= 16.858\ \mathrm{Nm} \end{aligned}$$

Mechanical work of the force $P$ along the whole path $l$ is

$$W_P = W_{P1} + W_{P2} = (1.607 + 16.858) = 18.466\ \mathrm{Nm}$$

**Solution of Exercise 3.12.2**

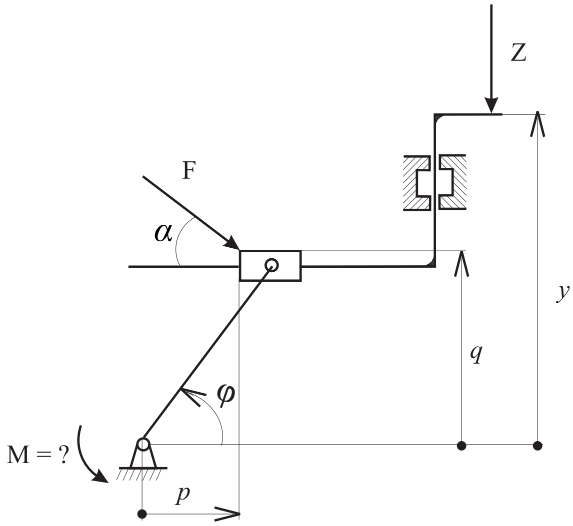


Figure 3.123: Exercise 3.12.2. Designation

The basic equation of pvw:

$$M\,\delta\varphi + F\,\cos\alpha\,\delta p - F\,\sin\alpha\,\delta q - Z\,\delta y = 0$$

Geometry:

$$\begin{aligned} p &= konst. + r\,\cos\varphi; & \delta p &= -r\,\sin\varphi\,\delta\varphi \\ q &= konst. + r\,\sin\varphi; & \delta q &= r\cos\varphi\,\delta\varphi \\ y &= konst. + r\,\sin\varphi; & \delta y &= r\,\cos\varphi\,\delta\varphi \end{aligned}$$

After substitution we have

$$M\,\delta\varphi - F\,\cos\alpha\,r\,\sin\varphi\,\delta\varphi - F\,\sin\alpha\,r\,\cos\varphi\,\delta\varphi - Z\,r\,\cos\varphi\,\delta\varphi = 0$$

Hence

$$M = r\left[F\left(\cos\alpha\,\sin\varphi + \sin\alpha\,\cos\varphi\right) + Z\,\cos\varphi\right]$$

or

$$M = r\left[F\,\sin(\alpha+\varphi) + Z\,\cos\varphi\right]$$

Result

$$M = 42.8\ \mathrm{Nm}$$

**Solution of Exercise 3.12.3**

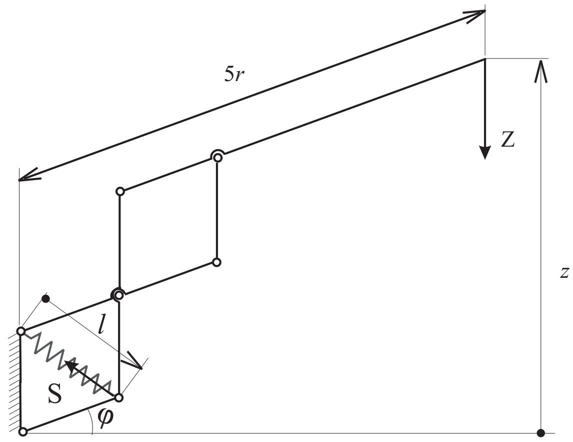


Figure 3.124: Exercise 3.12.3. Designation

The basic equation of pvw:

$$-S\,\delta l - Z\,\delta z = 0$$

Geometry:

$$\begin{aligned} l &= 2\,r\,\sin\frac{\frac{\pi}{2}-\varphi}{2}; & \delta l &= -r\,\cos(\tfrac{\pi}{4}-\tfrac{\varphi}{2})\,\delta\varphi \\ z &= \text{konst.} + 5\,r\,\sin\varphi; & \delta z &= 5\,r\,\cos\varphi\,\delta\varphi \end{aligned}$$

After substitution we have:

$$S\,r\,\cos(\frac{\pi}{4}-\frac{\varphi}{2})\,\delta\varphi - Z\,5\,r\,\cos\varphi\,\delta\varphi = 0$$

Hence

$$S = \frac{5\,\cos\varphi}{\cos\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)}\,Z = \frac{5\,\cos 306^\circ}{\cos 30^\circ}\,50 = 250\ \mathrm{N}$$

The force in the spring:

$$S = k\,(2\,r\,\sin 30^\circ - l_0)$$

The stiffness:

$$k = \frac{250}{2\,0.1\,\sin 30^\circ - 0.07} = 8333.3\ \mathrm{Nm}^{-1}$$

**Solution of Exercise 3.12.4**

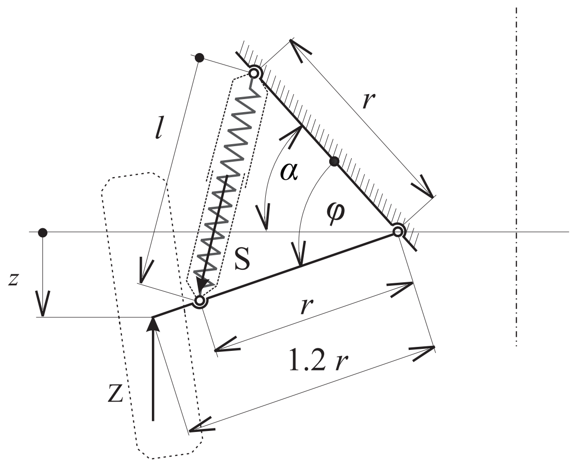


Figure 3.125: Exercise 3.12.4. Designation

The basic equation of pvw:

$$-Z\,\delta z + S\,\delta l = 0$$

Geometry:

$$\begin{array}{lcllcl} z &=& 1.2\,r\,\sin(\varphi-\alpha); & \delta z &=& 1.2\,r\,\cos(\varphi-\alpha)\,\delta\varphi \\ l &=& 2\,r\,\sin\frac{\varphi}{2}; & \delta l &=& 2\,r\,\cos\frac{\varphi}{2}\,\frac{1}{2}\,\delta\varphi \end{array}$$

After substitution we have

$$S = \frac{\delta z}{\delta l}\,Z = \frac{l.2\,r\,\cos(\varphi-\alpha)\,\delta\varphi}{2\,r\,\cos\frac{\varphi}{2}\,\frac{1}{2}\,\delta\varphi}\,Z$$

Hence

$$S = \frac{1.2\,\cos(\varphi-\alpha)}{\cos\frac{\varphi}{2}}\,Z$$

$$S = \frac{1.2\cos 5^\circ}{\cos 30^\circ}\,2500 = 3450.92\ \mathrm{N}$$

The force in the spring is

$$S = k\,\xi = k\ 0.1$$

Result:

$$k = 10\,S = 34509.2\ \mathrm{Nm}^{-1}$$

**Solution of Exercise 3.12.5**

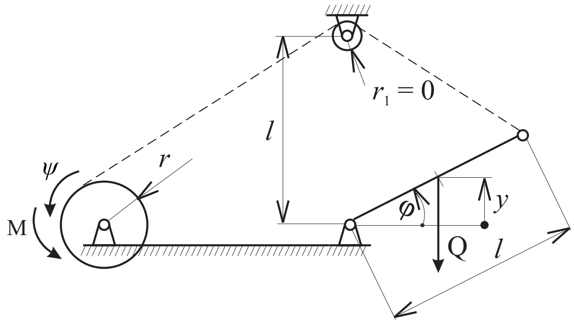


Figure 3.126: Exercise 3.12.5. Designation

The basic equation of pvw:

$$-Q\,\delta y + M\,\delta\psi = 0$$

Geometry:

$$\begin{aligned} y &= \tfrac{l}{2}\sin\varphi; & \delta y &= \tfrac{l}{2}\cos\,\delta\varphi \\ r\,\psi &= l\sqrt{2} - 2\,l\,\sin\tfrac{\frac{\pi}{2}-\varphi}{2}; & \delta\psi &= \tfrac{l}{r}\cos(\tfrac{\pi}{4} - \tfrac{\varphi}{2})\,\delta\varphi \end{aligned}$$

After substitution we have

$$-Q\frac{l}{2}\cos\varphi\,\delta\varphi + M\frac{l}{r}\cos(\frac{\pi}{4} - \frac{\varphi}{2})\,\delta\varphi = 0$$

Result:

$$M = \frac{r}{2}\,\frac{\cos\varphi}{\cos(\frac{\pi}{4} - \frac{\varphi}{2})}\,Q = \frac{0.1}{2}\,\frac{\cos 30^\circ}{\cos 30^\circ}\,5000 = 250\ \mathrm{Nm}$$

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