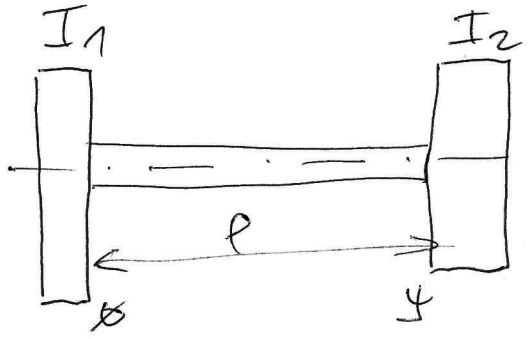
⑦



Torzní kmity

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{G}{\varrho} \frac{\partial^2 \xi}{\partial x^2}$$

Rozdělení na části

$\xi(x,t) = X(x)\,T(t)$

$\xi(\varphi,t) = \Phi(\varphi)\,T(t)$

$(M_2 = I\ddot{\varphi})$ $M_t = G J_p \frac{\partial \xi}{\partial x}$

$(\tau = E \cdot \varepsilon)$

$\Phi(\varphi) = C\cos\alpha\varphi + D\sin\alpha\varphi$

$\xi(\varphi,t) = \Phi(\varphi)\,T(t)$

$\dot{\xi} = \Phi(\varphi)\,\dot{T}(t)$

$\ddot{\xi} = \Phi(\varphi)\,\ddot{T}(t) \Rightarrow = -\Omega^2 \Phi(\varphi)\,T(t)$

$\alpha = \frac{\Omega_0}{c_0}$ $c_0 = \sqrt{\frac{G}{\varrho}}$

$$M_1 = I_1 \frac{\partial^2 \xi(0,t)}{\partial t^2} = I_1 \left[-\Omega^2 (C\cos\alpha\cdot 0 + D\sin\alpha\cdot 0)\right]$$

$$M_2 = I_2 \frac{\partial^2 \xi(\varphi,t)}{\partial t^2} = I_2 \left[-\Omega^2 (C\cos\alpha\cdot\varphi + D\sin\alpha\varphi)\right]$$

$$M_1 = G J_p \frac{\partial \xi}{\partial \varphi} = G J_p \cdot \Phi'(0)\,T(t)$$

$$M_2 = G J_p \frac{\partial \xi}{\partial \varphi} = G J_p\, \Phi'(\varphi)\,T(t)$$

$$\Phi = C\cos\alpha\cdot\varphi + D\cdot\sin\alpha\cdot\varphi$$

$$\Phi' = -C\alpha\sin\alpha\varphi + D\alpha\cos\alpha\varphi$$

(2)

Rozwiąże

$$M_1 = G J_p \left[-C\alpha \sin\alpha\varphi + D\alpha \underbrace{\cos\alpha\varphi}_{1} \right] =$$

$$= I_1 \left[-\Omega^2 \left(C \underbrace{\cos\alpha\varphi}_{1} + D \sin\alpha\varphi \right) \right]$$

$$G J_p D\alpha = -I_1 \Omega^2 C$$

$$M_2 = G J_p \left[-C\alpha \sin\alpha\psi + D\alpha\cos\alpha\psi \right] =$$

$$= I_2 \left[-\Omega^2 (C\cos\alpha\psi + D\sin\alpha\psi) \right]$$

$$\left[\begin{array}{c|c} I_1\Omega^2 & G J_p \alpha \\ \hline -G J_p \alpha \sin\alpha\psi + & G J_p \alpha \cos\alpha\psi \\ +I_2\Omega^2\cos\alpha\psi & -I_2\Omega^2\sin\alpha\psi \end{array}\right] \left[\begin{array}{c} C \\ \hline D \end{array}\right] = \left[\begin{array}{c} \not{0} \\ \hline \not{0} \end{array}\right]$$