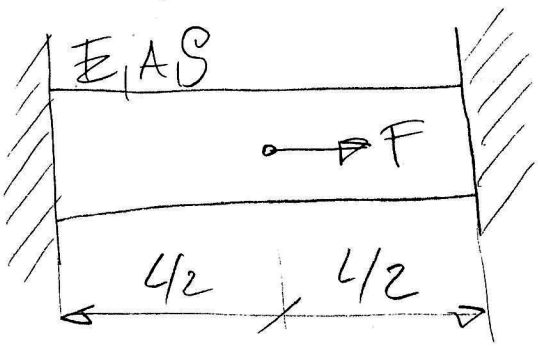


D: E, A, S, L, F

U: Změny po odstranění síly F

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{E}{S}\frac{\partial^2 \xi}{\partial x^2} \quad \leftrightarrow \quad \xi(x,t) = X(x)T(t)$$

$$\xi(x,t) = \sum_{n=1}^{\infty}\left[C_n \cos \alpha x + D_n \sin \alpha x\right]\left[A_n^* \cos \omega_0 t + B_n^* \sin \omega_0 t\right]$$

$$\alpha = \frac{\pi \cdot n}{L}$$

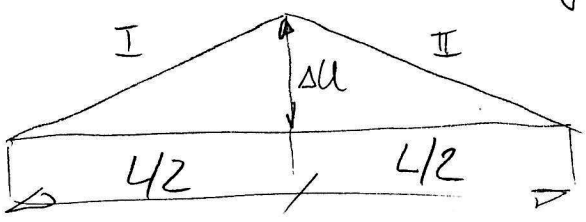
$$\xi(x,t) = \sum_{n=1}^{\infty}\left[A_n^* \cos \omega_0 t + B_n^* \sin \omega_0 t\right] D_n \sin\left(\frac{\pi n}{L} \cdot x\right)$$

Do závislosti do A_n a B_n

$$\xi(x,t) = \sum_{n=1}^{\infty}\left[A_n \cos \omega_0 t + B_n \sin \omega_0 t\right] \sin\left(\frac{\pi n}{L} x\right) = X(x)T(t)$$

A_n, B_n z počátečních podmínek

deformace prvku



$$\sigma = E \cdot \varepsilon = \frac{F}{A} = E \cdot \frac{\Delta u}{\frac{L}{2}}$$

$$\Rightarrow \Delta u = \frac{F \cdot L}{2AE}$$

$$^{I}u(x) = \frac{\Delta u}{L/2} \cdot x = \frac{\frac{F \cdot L}{2AE}}{\frac{L}{2}} \cdot x = \frac{F}{EA} x \qquad ^{I}x \in \langle 0; L/2 \rangle$$

$$^{II}u(x) = \frac{F}{EA}(L - x) \qquad ^{II}x \in \langle L/2; L \rangle$$

Počáteční podmínky

funkce rychlosti po celé délce $= 0$ v čase $t = 0$

$$\dot{\xi}(x,t) = \sum_{n=1}^{\infty} \underbrace{[-A_n \omega_{0n} \sin \omega_{0n} t}_{t=0 \Rightarrow 0} + \underbrace{B_n \omega_{0n} \cos \omega_{0n} t}_{t=0 \Rightarrow 1}] \sin\left(\frac{\pi n}{L} x\right) = 0$$

$$B_n \omega_{0n} \sin\left(\frac{\pi n}{L} x\right) = 0 \Rightarrow \underline{B_n = 0}$$

$$\Rightarrow \xi(x,t) = \sum_{n=1}^{\infty} A_n \cos \omega_{0n} t \sin\left(\frac{\pi n}{L} x\right)$$

dosadíme poč. p., že $t = 0$

$$\xi(x,0) = \sum_{n=1}^{\infty} A_n \underbrace{\sin\left(\frac{\pi n}{L} x\right)}_{\Phi_n} = A_1 \underbrace{\sin\left(\frac{\pi}{L} x\right)}_{\Phi_1} + A_2 \underbrace{\sin\left(\frac{2\pi}{L} x\right)}_{\Phi_2} \cdots$$

jízdní funkce „t"

$$\xi(x,0) = A_1 \sin\left(\frac{\pi}{L} x\right) + A_2 \sin\left(\frac{2\pi}{L} x\right) \cdots \quad / \cdot \Phi_n\left(\frac{\pi n}{L} x\right)$$

(využijeme ortogonalitu vl. tvarů Eulera)

$$\int \xi(x,0) \Phi_n\left(\frac{\pi n}{L} x\right) dx = A_n \int \underbrace{\Phi_n\left(\frac{\pi n}{L} x\right) \Phi_n\left(\frac{\pi n}{L} x\right) dx}_{\neq 0}$$

$$\int \Phi_i(x) \Phi_j(x) dx = 0$$

$$A_n = \frac{\int \xi(x,0)\,\Phi_n\left(\frac{\pi n}{L}x\right)dx}{\int \Phi_n\left(\frac{\pi n}{L}x\right)\Phi_n\left(\frac{\pi n}{L}x\right)} \quad ; \quad \Phi_n = \sin\left(\frac{\pi n}{L}x\right)$$

Počáteční okrajové podmínky

$$\xi(x,0) = \begin{cases} {}^{I}u(x) & x \in \langle 0, L/2\rangle \\ {}^{II}u(x) & x \in \langle L/2; L\rangle \end{cases} \qquad {}^{I}u(x) = \frac{F}{EA}\cdot x$$

$${}^{II}u(x) = \frac{F}{EA}(L-x)$$

$$A_n = \frac{\int_0^{L/2} \frac{F}{EA} x \sin\left(\frac{\pi n}{L}x\right)dx + \int_{L/2}^{L} \frac{F}{EA}(L-x)\sin\left(\frac{\pi n}{L}x\right)dx}{\int_0^L \sin^2\left(\frac{\pi n}{L}x\right)dx}$$

Pozn. 1 ⇓ $\int_0^L \sin\left(\frac{j\pi}{L}x\right)\sin\left(\frac{k\pi}{L}x\right)dx$ < $\frac{L}{2}$ $j=k$; $\emptyset$ $j\neq k$

Pozn. 2 $\int x\sin(cx)dx = \frac{\sin(cx)}{c^2} - \frac{x\cos(cx)}{c}$

$\Rightarrow \underline{\underline{A_n}}$

$B_n = \emptyset$

$C_n \to \emptyset$

$D_n \to$ zahrnuté do A_n, B_n

$\Big\}\ \xi(x,t) = \dots$