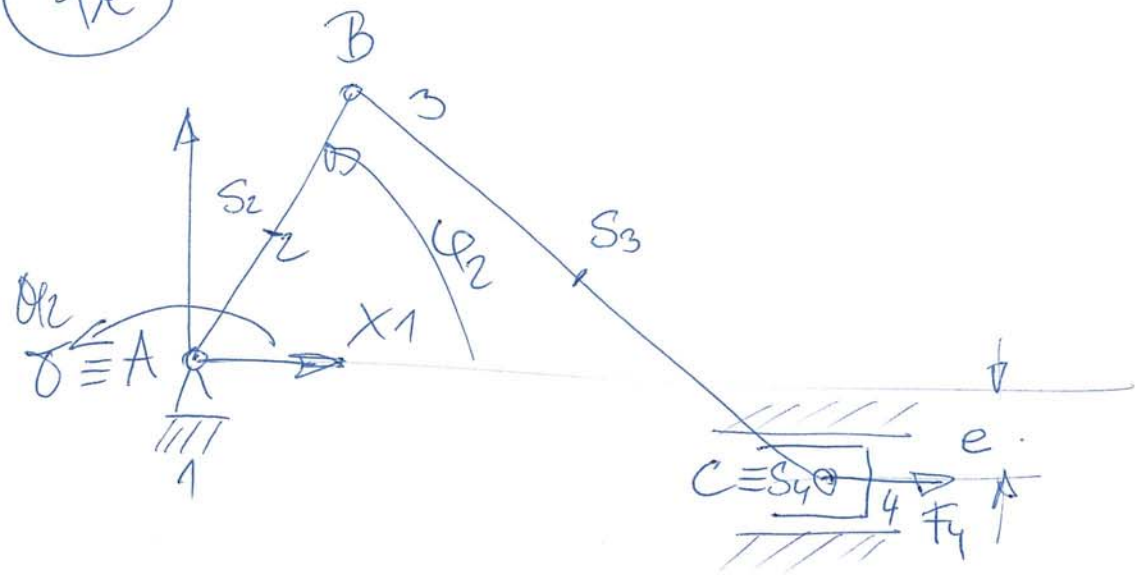
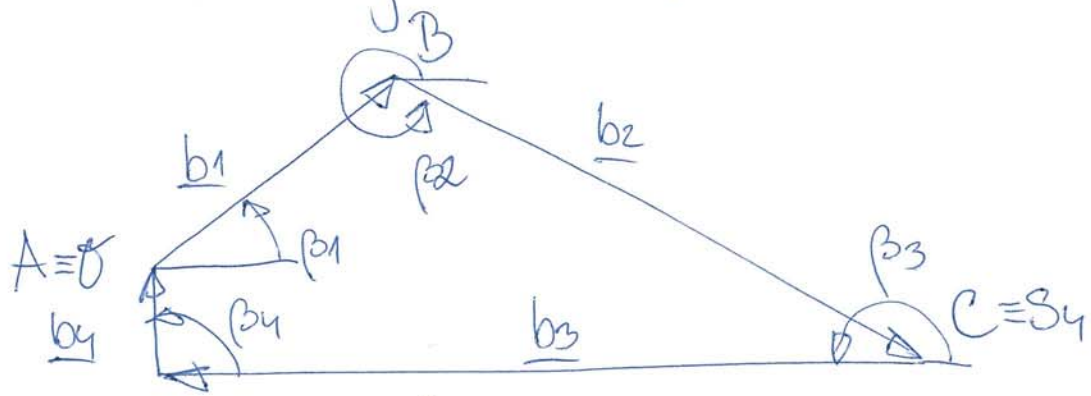
Př.



1) $i = 3(4-1) - 3_{rot} \cdot 2 - 1_{pos} \cdot 2 = 9 - 6 - 2 = 1^{\circ}$ volnosti

$\Rightarrow$ 1 nezávislá souřadnice
(např. úhel natočení členu)

2) Vektorový mnohoúhelník



Vektorová smyčka

$\underline{b_1} + \underline{b_2} + \underline{b_3} + \underline{b_4} = \underline{0}$

Skalární podmínky

x: $b_1 \cos\beta_1 + b_2 \cos\beta_2 + b_3 \cos\beta_3 + b_4 \cos\beta_4 = 0$

y: $b_1 \sin\beta_1 + b_2 \sin\beta_2 + b_3 \sin\beta_3 + b_4 \sin\beta_4 = 0$

b_i	delta	β_i	ideal
b_1	l_2	β_1	q_1
b_2	l_3	β_2	z_1
b_3	z_2	β_3	π
b_4	c	β_4	$\pi/2$

$AB = l_2$

$BC = l_3$

$$\underline{q} = [\beta_1]$$

$$\underline{z} = \begin{bmatrix} \beta_2 \\ b_3 \end{bmatrix}$$

1. derivace

$\underline{\dot{x}}$: $-b_1\dot{\beta}_1 \sin\beta_1 - b_2\dot{\beta}_2 \sin\beta_2 + \dot{b}_3 \cos\beta_3 = 0$

$\underline{\dot{y}}$: $b_1\dot{\beta}_1 \cos\beta_1 + b_2\dot{\beta}_2 \cos\beta_2 + \dot{b}_3 \sin\beta_3 = 0$

$$\begin{bmatrix} -b_1\sin\beta_1 \\ b_1\cos\beta_1 \end{bmatrix} [\dot{\beta}_1] + \begin{bmatrix} -b_2\sin\beta_2 & \cos\beta_3 \\ b_2\cos\beta_2 & \sin\beta_3 \end{bmatrix} \begin{bmatrix} \dot{\beta}_2 \\ \dot{b}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\underline{J_q} \cdot \underline{\dot{q}} + \underline{J_z} \cdot \underline{\dot{z}} = \underline{0}$$

$\ddot{x}$: $-b_1\ddot{\beta}_1 \sin\beta_1 - b_1\dot{\beta}_1^2 \cos\beta_1 - b_2\ddot{\beta}_2 \sin\beta_2 - b_2\dot{\beta}_2^2 \cos\beta_2 + b_3\ddot{\beta}_3 \cos\beta_3 = 0$

$\ddot{y}$: $b_1\ddot{\beta}_1 \cos\beta_1 - b_1\dot{\beta}_1^2 \sin\beta_1 + b_2\ddot{\beta}_2 \cos\beta_2 - b_2\dot{\beta}_2^2 \sin\beta_2 + b_3\ddot{\beta}_3 \sin\beta_3 = 0$

$$\begin{bmatrix} -b_1\sin\beta_1 \\ b_1\cos\beta_1 \end{bmatrix} [\ddot{\beta}_1] + \begin{bmatrix} -b_2\sin\beta_2 & \cos\beta_3 \\ b_2\cos\beta_2 & \sin\beta_3 \end{bmatrix} \begin{bmatrix} \ddot{\beta}_2 \\ \ddot{b}_3 \end{bmatrix} +$$

$$\begin{bmatrix} -b_1\dot{\beta}_1^2\cos\beta_1 - b_2\dot{\beta}_2^2\cos\beta_2 \\ -b_1\dot{\beta}_1^2\sin\beta_1 - b_2\dot{\beta}_2^2\sin\beta_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$J_q \cdot \ddot{q} + J_z \cdot \ddot{z} + j_{qz} = 0$$

$r_{1S2} = X_{S2}$

$r_{1S3} = b_1 + X_{S3}$

$r_{1S4} = b_1 + b_2$

$x_{1S2} = x_{S2} \cos\beta_1$

$y_{1S2} = x_{S2} \sin\beta_1$

$x_{1S3} = b_1 \cos\beta_1 + x_{S3} \cos\beta_2$

$y_{1S3} = b_1 \sin\beta_1 + x_{S3} \sin\beta_2$

$x_{1S4} = b_1 \cos\beta_1 + b_2 \cos\beta_2$

$y_{1S4} = b_1 \sin\beta_1 + b_2 \sin\beta_2$

$\dot{x}_{1S2} = -x_{S2} \sin\beta_1 \cdot \dot{\beta}_1$

$\ddot{x}_{1S2} = -x_{S2}\ddot{\beta}_1 \sin\beta_1 - x_{S2}\dot{\beta}_1^2 \cos\beta_1$

$\dot{y}_{1S2} = x_{S2}\dot{\beta}_1 \cos\beta_1$

$\ddot{y}_{1S2} = x_{S2}\ddot{\beta}_1 \cos\beta_1 - x_{S2}\dot{\beta}_1^2 \sin\beta_1$

$$\dot{x}_{1S3} = -b_1\dot{\beta}_1 \sin\beta_1 - x_{S3}\dot{\beta}_2 \sin\beta_2$$

$$\ddot{x}_{1S3} = -b_1\ddot{\beta}_1 \sin\beta_1 - b_1\dot{\beta}_1^2 \cos\beta_1 - x_{S3}\ddot{\beta}_2 \sin\beta_2 - x_{S3}\dot{\beta}_2^2 \cos\beta_2$$

$$\dot{y}_{1S3} = b_1\dot{\beta}_1 \cos\beta_1 + x_{S3}\dot{\beta}_2 \cos\beta_2$$

$$\ddot{y}_{1S3} = b_1\ddot{\beta}_1 \cos\beta_1 - b_1\dot{\beta}_1^2 \sin\beta_1 + x_{S3}\ddot{\beta}_2 \cos\beta_2 - x_{S3}\dot{\beta}_2^2 \sin\beta_2$$

$$\dot{x}_{1S4} = -b_1\dot{\beta}_1 \sin\beta_1 - b_2\dot{\beta}_2 \sin\beta_2$$

$$\ddot{x}_{1S4} = -b_1\ddot{\beta}_1 \sin\beta_1 - b_1\dot{\beta}_1^2 \cos\beta_1 - b_2\ddot{\beta}_2 \sin\beta_2 - b_2\dot{\beta}_2^2 \cos\beta_2$$

$$\dot{y}_{1S4} = b_1\dot{\beta}_1 \cos\beta_1 + b_2\dot{\beta}_2 \cos\beta_2$$

$$\ddot{y}_{1S4} = b_1\ddot{\beta}_1 \cos\beta_1 - b_1\dot{\beta}_1^2 \sin\beta_1 + b_2\ddot{\beta}_2 \cos\beta_2 - b_2\dot{\beta}_2^2 \sin\beta_2$$

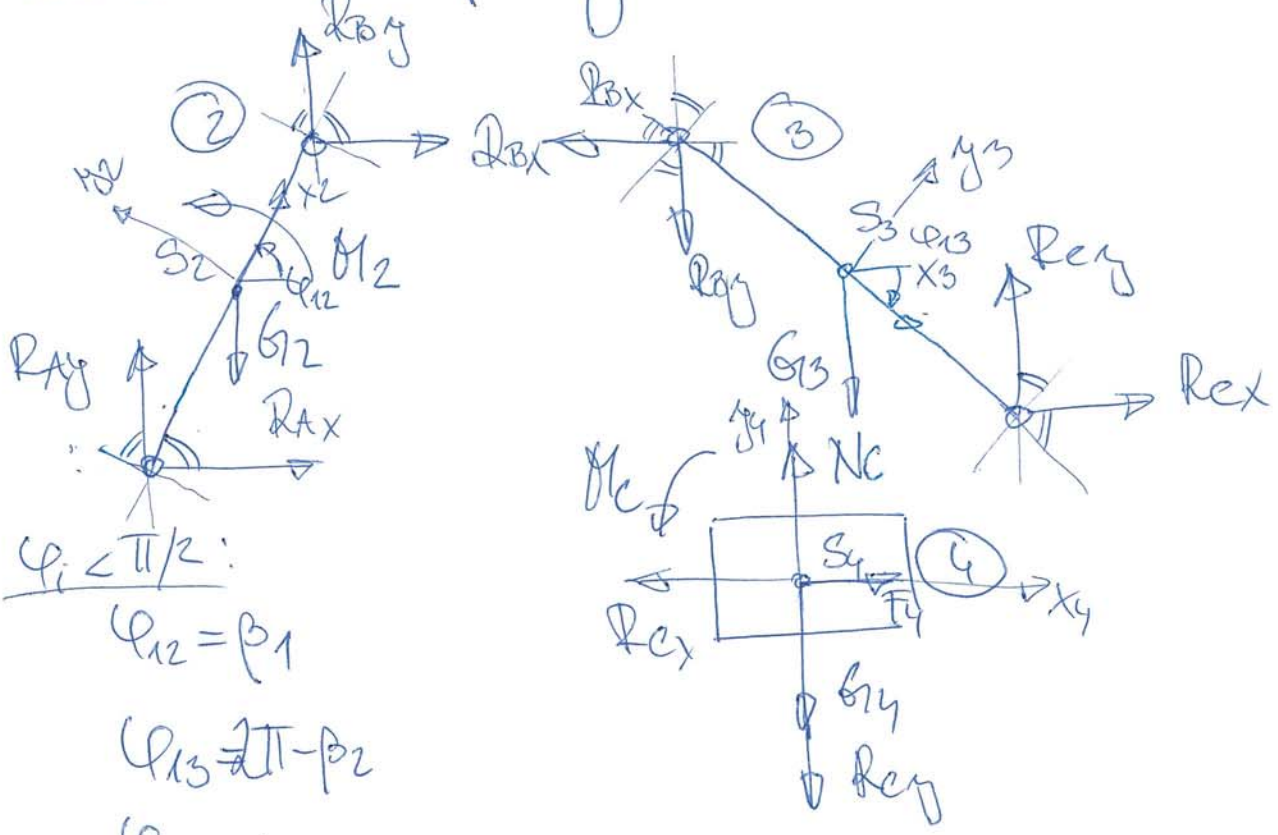
Kinematika středu hmotnosti mat. ( )

$$\begin{bmatrix} \ddot{x}_{1S2} \\ \ddot{y}_{1S2} \\ \ddot{\varphi}_{12} \\ \ddot{x}_{1S3} \\ \ddot{y}_{1S3} \\ \ddot{\varphi}_{13} \\ \ddot{x}_{1S4} \\ \ddot{y}_{1S4} \\ \ddot{\varphi}_{14} \end{bmatrix} = \begin{bmatrix} -x_{S2}\sin\beta_1 \\ x_{S2}\cos\beta_1 \\ 1 \\ -b_1\sin\beta_1 \\ b_1\cos\beta_1 \\ 0 \\ -b_1\sin\beta_1 \\ b_1\cos\beta_1 \\ 0 \end{bmatrix} \begin{bmatrix} \ddot{\beta}_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -x_{S3}\sin\beta_2 & 0 \\ x_{S3}\cos\beta_2 & 0 \\ -1 & \\ -b_2\sin\beta_2 & 0 \\ b_2\cos\beta_2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{\beta}_2 \\ \ddot{b}_3 \end{bmatrix} +$$

$$+ \begin{bmatrix} -x_{S2}\dot{\beta}_1^2\cos\beta_1 \\ -x_{S2}\dot{\beta}_1^2\sin\beta_1 \\ 0 \\ -x_{S3}\dot{\beta}_2^2\cos\beta_2 - b_1\dot{\beta}_1^2\cos\beta_1 \\ -b_1\dot{\beta}_1^2\sin\beta_1 - x_{S3}\dot{\beta}_2^2\sin\beta_2 \\ 0 \\ -b_1\dot{\beta}_1^2\cos\beta_1 - b_2\dot{\beta}_2^2\cos\beta_2 \\ -b_1\dot{\beta}_1^2\sin\beta_1 - b_2\dot{\beta}_2^2\sin\beta_2 \\ 0 \end{bmatrix}$$

$$\underline{a} = \underline{V_q}\,\ddot{\underline{q}} + \underline{V_t}\,\ddot{\underline{t}} + \underline{a_{qt}}$$

Механизм - рабочая точка



$\varphi_i < \pi/2$:

$\varphi_{12} = \beta_1$

$\varphi_{13} = 2\pi - \beta_2$

$\varphi_{14} = \emptyset$

$\ddot{\varphi}_{12} = \ddot{\beta}_1 \; ; \ddot{\varphi}_{13} = -\ddot{\beta}_2 \; ; \ddot{\varphi}_{14} = \emptyset$

(2)

$$m_2 \ddot{x}_{1S_2} = R_{Ax} + R_{Bx}$$

$$m_2 \ddot{y}_{1S_2} = R_{Ay} + R_{By} - G_{12}$$

$$I_{2S_2} \ddot{\varphi}_{12} = M_2 + R_{Ax} x_{S_2} \sin\varphi_{12} - R_{Ay} x_{S_2} \cos\varphi_{12} - R_{Bx} (l_2 - x_{S_2}) \sin\varphi_{12} + R_{By} (l_2 - x_{S_2}) \cos\varphi_{12}$$

(3)

$$m_3 \ddot{x}_{1S_3} = R_{cx} - R_{Bx}$$

$$m_3 \ddot{y}_{1S_3} = R_{cy} - R_{By} - G_{13}$$

$$I_{3S_3} \ddot{\varphi}_{13} = -R_{Bx} \sin\varphi_{13} x_{S_3} - R_{By} x_{S_3} \cos\varphi_{13} - R_{cx} (l_3 - x_{S_3}) \sin\varphi_{13} - R_{cy} (l_3 - x_{S_3}) \cos\varphi_{13}$$

(4)

$$m_4 \ddot{x}_{1S4} = -R_{ex} + F_4$$

$$m_4 \ddot{y}_{1S4} = N_c - G_4 - R_{ey}$$

$$I_{1S4} \ddot{\varphi}_{14} = M_c$$

$$\underline{M_a} = \underline{Q} + \underline{D} \cdot \underline{R}$$

Maticový zápis pohybových rovnic:

$$\begin{bmatrix} m_2 & & & & & & & & \\ & m_2 & & & & & & & \\ & & I_{2S2} & & & & & & \\ & & & m_3 & & & & & \\ & & & & m_3 & & & & \\ & & & & & I_{3S3} & & & \\ & & & & & & m_4 & & \\ & & & & & & & m_4 & \\ & & & & & & & & I_{1S4} \end{bmatrix} \begin{bmatrix} \ddot{x}_{1S2} \\ \ddot{y}_{1S2} \\ \ddot{\varphi}_{12} \\ \ddot{x}_{1S3} \\ \ddot{y}_{1S3} \\ \ddot{\varphi}_{13} \\ \ddot{x}_{1S4} \\ \ddot{y}_{1S4} \\ \ddot{\varphi}_{14} \end{bmatrix} = \begin{bmatrix} 0 \\ -G_2 \\ M_2 \\ 0 \\ -G_3 \\ 0 \\ F_4 \\ -G_4 \\ 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} 1 & & 1 & & & & & \\ & 1 & & 1 & & & & \\ x_{2S}\varphi_{12} & -x_{S2}c\varphi_{12} & -(l_2-x_{S2})s\varphi_{12} & (l_2-x_{S2})c\varphi_{12} & & & & \\ & & -1 & & 1 & & & \\ & & & -1 & & 1 & & \\ & & x_{S3}s\varphi_{12} & -x_{S3}c\varphi_{12} & -(l_3-x_{S3})s\varphi_{12} & -(l_3-x_{S3})c\varphi_{13} & & \\ & & & & -1 & & & \\ & & & & & -1 & 1 & \\ & & & & & & & 1 \end{bmatrix} \begin{bmatrix} R_{AX} \\ R_{AY} \\ R_{BX} \\ R_{BY} \\ R_{ex} \\ R_{ey} \\ N_c \\ M_c \end{bmatrix}$$

Dostaneme tedy tři maticové rovnice pro zrychlení a poh. rovnice ( )

$$\underline{\varnothing} = \underline{J_q}\cdot\ddot{\underline{q}} + \underline{J_z}\cdot\ddot{\underline{z}} + \underline{j_{qz}}$$

$$\underline{a} = \underline{V_q}\cdot\ddot{\underline{q}} + \underline{V_z}\ddot{\underline{z}} + \underline{a_{qz}}$$

$$\underline{M}\,\underline{a} = \underline{Q} + \underline{D}\cdot\underline{R}$$

$$\Downarrow$$

$$\left[\begin{array}{c|c|c|c} \underline{M} & -\underline{D} & \underline{\varnothing} & \underline{\varnothing} \\ \hline \underline{T} & \underline{\varnothing} & -\underline{V_z} & -\underline{V_q} \\ \hline \underline{\varnothing} & \underline{\varnothing} & \underline{J_z} & \underline{J_q} \end{array}\right] \left[\begin{array}{c} \underline{a} \\ \underline{R} \\ \ddot{\underline{z}} \\ \ddot{\underline{q}} \end{array}\right] = \left[\begin{array}{c} \underline{Q} \\ \underline{a_{qz}} \\ -\underline{j_{qz}} \end{array}\right]$$