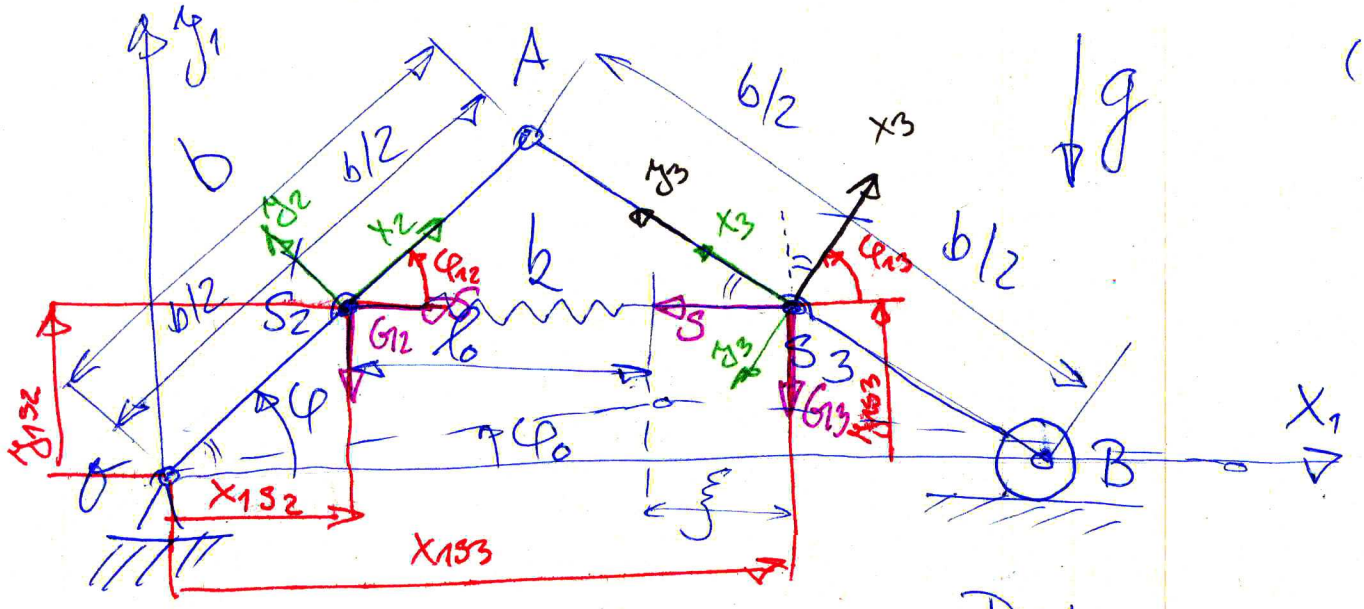

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}_i}\right) - \frac{\partial E_k}{\partial q_i} = Q_i$$

D: m_2, m_3

$I_{2O}, I_{3S3}, b,$

l_0, ξ, g

$$q = [\varphi_{12}]$$

$$s = [\varphi_{12}, x_{1S2}, y_{1S2}, x_{1S3}, y_{1S3}, \varphi_{13}]$$

$$E_k = \frac{1}{2} m_2 \cdot (\dot{x}_{1S2}^2 + \dot{y}_{1S2}^2) + \frac{1}{2} I_{2S2} \cdot \dot{\varphi}_{12}^2 +$$

$$+ \frac{1}{2} m_3 (\dot{x}_{1S3}^2 + \dot{y}_{1S3}^2) + \frac{1}{2} I_{3S3} \cdot \dot{\varphi}_{13}^2$$

$$x_{1S2} = \frac{b}{2} \cdot \cos\varphi_{12} \qquad y_{1S2} = \frac{b}{2} \sin\varphi_{12}$$

$$\dot{x}_{1S2} = -\frac{b}{2} \sin\varphi_{12} \cdot \dot{\varphi}_{12} \qquad \dot{y}_{1S2} = \frac{b}{2} \cos\varphi_{12} \cdot \dot{\varphi}_{12}$$

$$\left(\frac{b}{2} \cdot \cos\varphi_{12} \cdot \dot{\varphi}_{12}\right)^2 + \left(-\frac{b}{2} \sin\varphi_{12} \dot{\varphi}_{12}\right)^2 = \left(\frac{b}{2}\right)^2 \dot{\varphi}_{12}^2 \left(\cos^2\varphi_{12} + \sin^2\varphi_{12}\right) =$$

$$= \left(\frac{b}{2}\right)^2 \dot{\varphi}_{12}^2$$

$$x_{153} = b\cdot\cos\varphi_{12} + \frac{b}{2}\cos\varphi_{12} \qquad y_{153} = \frac{b}{2}\cdot\sin\varphi_{12}$$

$$= \frac{3}{2}b\cos\varphi_{12}$$

$$\dot{x}_{153} = -\frac{3}{2}b\sin\varphi_{12}\cdot\dot{\varphi}_{12} \qquad \dot{y}_{153} = \frac{b}{2}\cos\varphi_{12}\cdot\dot{\varphi}_{12}$$

$$\dot{x}_{153}^2 + \dot{y}_{153}^2 =$$

$$\left(\frac{3}{2}b\right)^2\sin^2\varphi_{12}\cdot\dot{\varphi}_{12}^2 + \left(\frac{b}{2}\right)^2\cos^2\varphi_{12}\dot{\varphi}_{12}^2$$

$$= \dot{\varphi}_{12}^2\left[\left(\frac{3}{2}b\right)^2\sin^2\varphi_{12} + \left(\frac{b}{2}\right)^2\cos^2\varphi_{12}\right]$$

$$= \dot{\varphi}_{12}^2 b^2\left[\left(\frac{3}{2}\right)^2\sin^2\varphi_{12} + \left(\frac{1}{2}\right)^2\cos^2\varphi_{12}\right]$$

$$\varphi_{13} = \pi/2 - \varphi_{12}$$

$$\dot{\varphi}_{13} = -\dot{\varphi}_{12}$$

$$T_2(\varphi_{12},\dot{\varphi}_{12}) = \frac{1}{2}m_2\left(\frac{b}{2}\right)^2\dot{\varphi}_{12}^2 + \frac{1}{2}I_{2S2}\dot{\varphi}_{12}^2 +$$

$$+ \frac{1}{2}m_3\dot{\varphi}_{12}^2 b^2\left[\left(\frac{3}{2}\right)^2\sin^2\varphi_{12} + \left(\frac{1}{2}\right)^2\cos^2\varphi_{12}\right] + \frac{1}{2}I_{3S3}\dot{\varphi}_{12}^2$$

$$T_2(\varphi_{12},\dot{\varphi}_{12}) = \frac{1}{2}\dot{\varphi}_{12}^2\Big\{\underbrace{I_{2S2} + m_2\left(\frac{b}{2}\right)^2}_{I_{20}} + m_3 b^2\left[\left(\frac{3}{2}\right)^2\sin^2\varphi_{12} + \left(\frac{1}{2}\right)^2\cos^2\varphi_{12}\right] + I_{3S3}\Big\}$$

$$\sum Q_i \delta s_i = -G_2 \delta y_{1S2} + S \cdot \delta x_{1S2} - G_3 \cdot \delta y_{1S3} - S \delta x_{1S3}$$

$$S = k \cdot \xi = k \cdot (x_{1S3} - l_0 - x_{1S2}) =$$

$$= k \left(\frac{3}{2} b \cdot \cos\varphi_{12} - l_0 - \frac{b}{2} \cos\varphi_{12} \right)$$

$$S = k (b \cos\varphi_{12} - l_0)$$

$$\sum Q_i \cdot \delta s_i = -G_2 \delta y_{1S2} + k(b\cos\varphi_{12} - l_0) \delta x_{1S2} -$$

$$- G_3 \delta y_{1S3} - k(b\cos\varphi_{12} - l_0) \delta x_{1S3}$$

$$y_{1S2} = \frac{b}{2} \sin\varphi_{12} \qquad x_{1S2} = \frac{b}{2} \cos\varphi_{12}$$

$$\delta y_{1S2} = \frac{b}{2} \cos\varphi_{12} \delta\varphi_{12} \qquad \delta x_{1S2} = -\frac{b}{2} \sin\varphi_{12} \delta\varphi_{12}$$

$$y_{1S3} = \frac{b}{2} \sin\varphi_{12} \qquad x_{1S3} = \frac{3}{2} b \cos\varphi_{12}$$

$$\delta y_{1S3} = \frac{b}{2} \cos\varphi_{12} \delta\varphi_{12} \qquad \delta x_{1S3} = -\frac{3}{2} b \sin\varphi_{12} \delta\varphi_{12}$$

$$Q \delta q = -G_2 \frac{b}{2} \cos\varphi_{12} \cdot \delta\varphi_{12} + k(b\cos\varphi_{12} - l_0)\left(-\frac{b}{2}\sin\varphi_{12}\right)\delta\varphi_{12} -$$

$$Q \cdot \delta\varphi_{12} - G_3 \frac{b}{2} \cos\varphi_{12} \delta\varphi_{12} + k(b\cos\varphi_{12} - l_0) \frac{3}{2} b \sin\varphi_{12} \delta\varphi_{12} =$$

$$Q \cdot \delta q \Rightarrow \delta q = \delta\varphi_{12}$$

$$\boxed{Q = \underbrace{-G_2 \frac{b}{2} \cos\varphi_{12} - G_3 \frac{b}{2} \cos\varphi_{12}}_{-(G_2+G_3)\frac{b}{2}\cos\varphi_{12}} + k(b\cos\varphi_{12} - l_0) b \sin\varphi_{12}}$$

$$\frac{\partial E_k}{\partial \dot{q}} = \frac{\partial E_k}{\partial \dot{\varphi}_{12}} =$$

$$= \dot{\varphi}_{12}\left\{ I_{20} + m_3 b^2 \left[\left(\frac{3}{2}\right)^2 \sin^2\varphi_{12} + \left(\frac{1}{2}\right)^2 \cos^2\varphi_{12} \right] + I_{333} \right\}$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}_{12}}\right) = \ddot{\varphi}_{12}\{\ \} + \dot{\varphi}_{12}\left\{ m_3 b^2 \left[\left(\frac{3}{2}\right)^2 \underbrace{2\cdot\sin\varphi_{12}\cdot\cos\varphi_{12}}_{\sin(2\varphi_{12})}\dot{\varphi}_{12} + \right.\right.$$

$$\left.\left. - \left(\frac{1}{2}\right)^2 \underbrace{2\cdot\cos\varphi_{12}\sin\varphi_{12}}_{\sin(2\varphi_{12})}\cdot\dot{\varphi}_{12} \right] \right\} \qquad \frac{9}{4} - \frac{1}{4} = \frac{8}{4} = 2$$

$$\Rightarrow 2\sin(2\varphi_{12})$$

$$\frac{\partial E_k}{\partial \varphi_{12}} = \frac{1}{2}\dot{\varphi}_{12}^2 \left\{ m_3 b^2 \left[\underbrace{\left(\frac{3}{2}\right)^2 2\sin\varphi_{12}\cdot\cos\varphi_{12} - \left(\frac{1}{2}\right)^2 2\cdot\cos\varphi_{12}\cdot\sin\varphi_{12}}_{2\sin(2\varphi_{12})} \right] \right\}$$

VPR

$$\ddot{\varphi}_{12}\left\{ I_{20} + m_3 b^2 \left[\left(\frac{3}{2}\right)^2 \sin^2\varphi_{12} + \left(\frac{1}{2}\right)^2 \cos^2\varphi_{12} \right] + I_{333} \right\} + \dot{\varphi}_{12}^2 \cdot m_3 b^2 \cdot 2\sin(2\varphi_{12}) -$$

$$- \frac{1}{2}\dot{\varphi}_{12}^2 \cdot 2 m_3 b^2 \sin(2\varphi_{12}) = -(G_2 + G_3)\frac{b}{2}\cos\varphi_{12} +$$

$$+ k(b\cos\varphi_{12} - l_0) b \sin\varphi_{12}$$