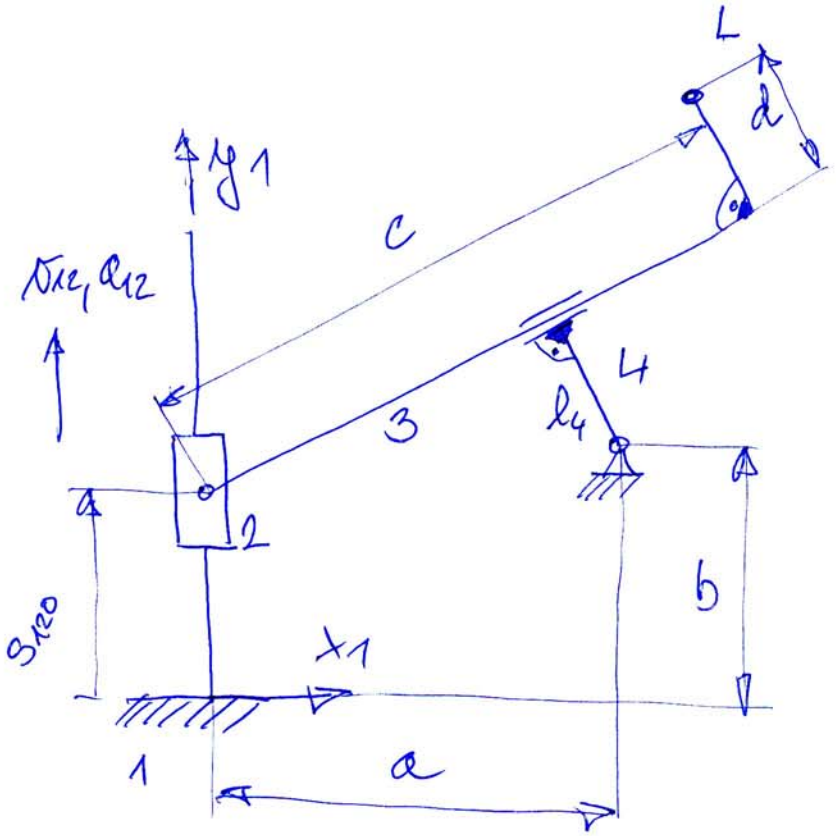


D: s_{120}, v_{12}, a_{12}
a, b, c, d

Počet stupňů volnosti

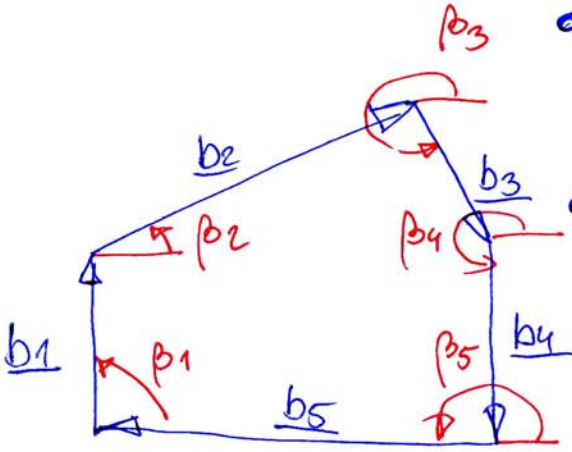
$i = 3(n-1) - 2 \cdot \text{rot} - 2 \cdot \text{pos.} = 3(4-1) - 2 \cdot 2 - 2 \cdot 2 = 1^\circ$ volnosti

Počet smyček

$s = d - n + m + 1 = 4 - 4 + 0 + 1 = 1$ smyčka

(čís. dvojic ← → těles předepš. pohybů)

Popis vektorové smyčky



- Nezávislé souřadnice

$q = [b_1] = s_{120} + v_{12}t + \frac{1}{2}a_{12}t^2$

- Závislé souřadnice

$z = \begin{bmatrix} b_2 \\ \beta_2 \end{bmatrix}$ $\beta_3 = \beta_2 + \frac{3}{2}\pi$

- Vektorová rovnice smyčky

$\underline{b_1} + \underline{b_2} + \underline{b_3} + \underline{b_4} + \underline{b_5} = \underline{0}$

Rozpis vektorové rovnice do složek

(složkových/skalárních rovnic)

POLOHA

$x: b_1 \cdot \cos\beta_1 + b_2\cos\beta_2 + b_3\cos\beta_3 + b_4\cos\beta_4 + b_5 \cdot \cos\beta_5 = 0$

$y: b_1 \cdot \sin\beta_1 + b_2 \cdot \sin\beta_2 + b_3\sin\beta_3 + b_4\sin\beta_4 + b_5 \sin\beta_5 = 0$

(soustava nelineárních rovnic)

RYCHLOST (1. derivace)

$$\dot{b}_1 \cos\beta_1 + \dot{b}_2 \cos\beta_2 - b_2 \sin\beta_2 \dot{\beta}_2 - b_3 \sin\beta_3 \cdot \dot{\beta}_3 = 0$$
$$\dot{b}_1 \sin\beta_1 + \dot{b}_2 \sin\beta_2 + b_2 \cos\beta_2 \dot{\beta}_2 + b_3 \cos\beta_3 \dot{\beta}_3 = 0$$

(soustava lineárních rovnic)

$\beta_3 = \beta_2 + \frac{3}{2}\pi$

$\dot{\beta}_3 = \dot{\beta}_2$

(proto je zahrnuto)

$$\underbrace{\begin{bmatrix} \cos\beta_1 \\ \sin\beta_1 \end{bmatrix}}_{J_q} \underbrace{\begin{bmatrix} \dot{b}_1 \end{bmatrix}}_{\dot{q}} + \underbrace{\begin{bmatrix} \cos\beta_2 & -b_2\sin\beta_2 - b_3\sin\beta_3 \\ \sin\beta_2 & b_2\cos\beta_2 + b_3\cos\beta_3 \end{bmatrix}}_{J_z} \underbrace{\begin{bmatrix} \dot{b}_2 \\ \dot{\beta}_2 \end{bmatrix}}_{\dot{z}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$J_q \cdot \dot{q} + J_z \cdot \dot{z} = 0$$

$$\hookrightarrow \dot{z} = -J_z^{-1} J_q \cdot \dot{q}$$

ZRYCHLENÍ (2. derivace)

$$\ddot{b}_1 \cos\beta_1 + \ddot{b}_2 \cos\beta_2 - \dot{b}_2 \sin\beta_2 \dot{\beta}_2 - \dot{b}_2 \sin\beta_2 \dot{\beta}_2 - b_2 \cos\beta_2 \cdot \dot{\beta}_2^2 - b_2 \cdot \sin\beta_2 \ddot{\beta}_2 - b_3 \cos\beta_3 \cdot \dot{\beta}_3^2 - b_3 \sin\beta_3 \cdot \ddot{\beta}_3 = 0$$

$$\ddot{b}_1 \sin\beta_1 + \ddot{b}_2 \sin\beta_2 + \dot{b}_2 \cos\beta_2 \dot{\beta}_2 + \dot{b}_2 \cos\beta_2 \dot{\beta}_2 - b_2 \sin\beta_2 \cdot \dot{\beta}_2^2 + b_2 \cdot \cos\beta_2 \cdot \ddot{\beta}_2 - b_3 \sin\beta_3 \cdot \dot{\beta}_3^2 + b_3 \cdot \cos\beta_3 \ddot{\beta}_3 = 0$$

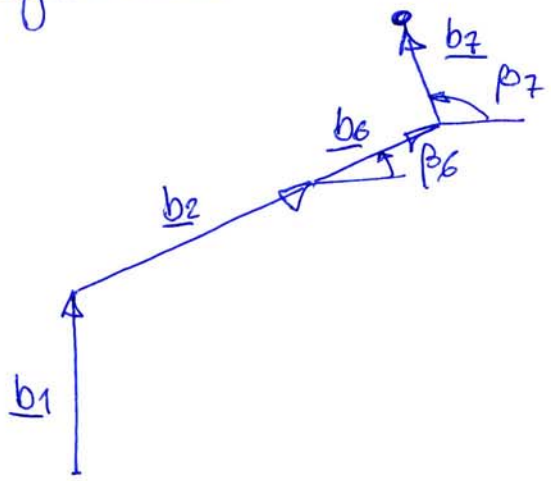
(soustava lineárních rovnic)

$$\underbrace{\begin{bmatrix} \cos\beta_1 \\ \sin\beta_1 \end{bmatrix}}_{J_q} \cdot \underbrace{\begin{bmatrix} \ddot{b}_1 \end{bmatrix}}_{\ddot{q}} + \underbrace{\begin{bmatrix} \cos\beta_2 & -b_2\sin\beta_2 - b_3\sin\beta_3 \\ \sin\beta_2 & b_2\cos\beta_2 + b_3\cos\beta_3 \end{bmatrix}}_{J_z} \underbrace{\begin{bmatrix} \ddot{b}_2 \\ \ddot{\beta}_2 \end{bmatrix}}_{\ddot{z}} + \underbrace{\begin{bmatrix} -2\dot{b}_2\dot{\beta}_2 \sin\beta_2 - b_2\cos\beta_2\dot{\beta}_2^2 - b_3\cos\beta_3\cdot\dot{\beta}_3^2 \\ 2\dot{b}_2\dot{\beta}_2\cos\beta_2 - b_2\sin\beta_2\dot{\beta}_2^2 - b_3\sin\beta_3\cdot\dot{\beta}_3^2 \end{bmatrix}}_{j_{qz}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$J_q \cdot \ddot{q} + J_z \cdot \ddot{z} + j_{qz} = 0$$

$$\Rightarrow \ddot{z} = -J_z^{-1}(J_q \cdot \ddot{q} + j_{qz})$$

Pohyb bodu L



$b_6 = c - b_2 \qquad \beta_6 = \beta_2$

$b_7 = d \qquad \beta_7 = \beta_2 + \frac{\pi}{2}$

$$\underline{r}_{1L} = \begin{bmatrix} x_{1L} \\ y_{1L} \end{bmatrix} = \underline{b}_1 + \underline{b}_2 + \underline{b}_6 + \underline{b}_7$$

$$x_{1L} = b_1 \cdot \cos\beta_1 + \underbrace{b_2\cos\beta_2 + b_6\cos\beta_6}_{c\cdot\cos\beta_2} + b_7\cos\beta_7$$

$$y_{1L} = b_1 \cdot \sin\beta_1 + \underbrace{b_2\sin\beta_2 + b_6\sin\beta_6}_{c\cdot\sin\beta_2} + b_7\sin\beta_7$$

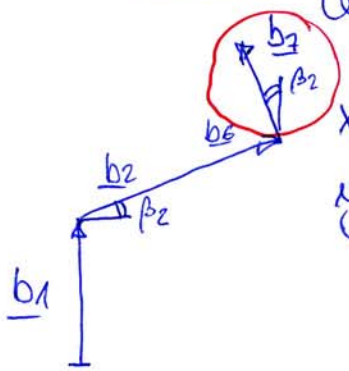
$$\dot{x}_{1L} = \dot{b}_1\cos\beta_1 - c\cdot\sin\beta_2\cdot\dot{\beta}_2 - d\cdot\sin\beta_7\cdot\dot{\beta}_7$$

$$\dot{y}_{1L} = \dot{b}_1\sin\beta_1 + c\cdot\cos\beta_2\cdot\dot{\beta}_2 + d\cos\beta_7\cdot\dot{\beta}_7$$

$$\ddot{x}_{1L} = \ddot{b}_1\cos\beta_1 - c\cdot\cos\beta_2\cdot\dot{\beta}_2^2 - c\cdot\sin\beta_2\ddot{\beta}_2 - d\cos\beta_7\dot{\beta}_7^2 - d\sin\beta_7\ddot{\beta}_7$$

$$\ddot{y}_{1L} = \ddot{b}_1\sin\beta_1 - c\cdot\sin\beta_2\cdot\dot{\beta}_2^2 + c\cos\beta_2\ddot{\beta}_2 - d\cdot\sin\beta_7\dot{\beta}_7^2 + d\cos\beta_7\cdot\ddot{\beta}_7$$

Poznámka: Pokud střídavě nedokážeme značení účelně, lze vektor vyjádřit i následovně:



$$x_{1L} = b_1\cdot\cos\beta_1 + b_2\cdot\cos\beta_2 + b_6\cos\beta_6 - d\cdot\sin\beta_2$$

$$y_{1L} = b_1\cdot\sin\beta_1 + b_2\sin\beta_2 + b_6\sin\beta_6 + d\cdot\cos\beta_2$$