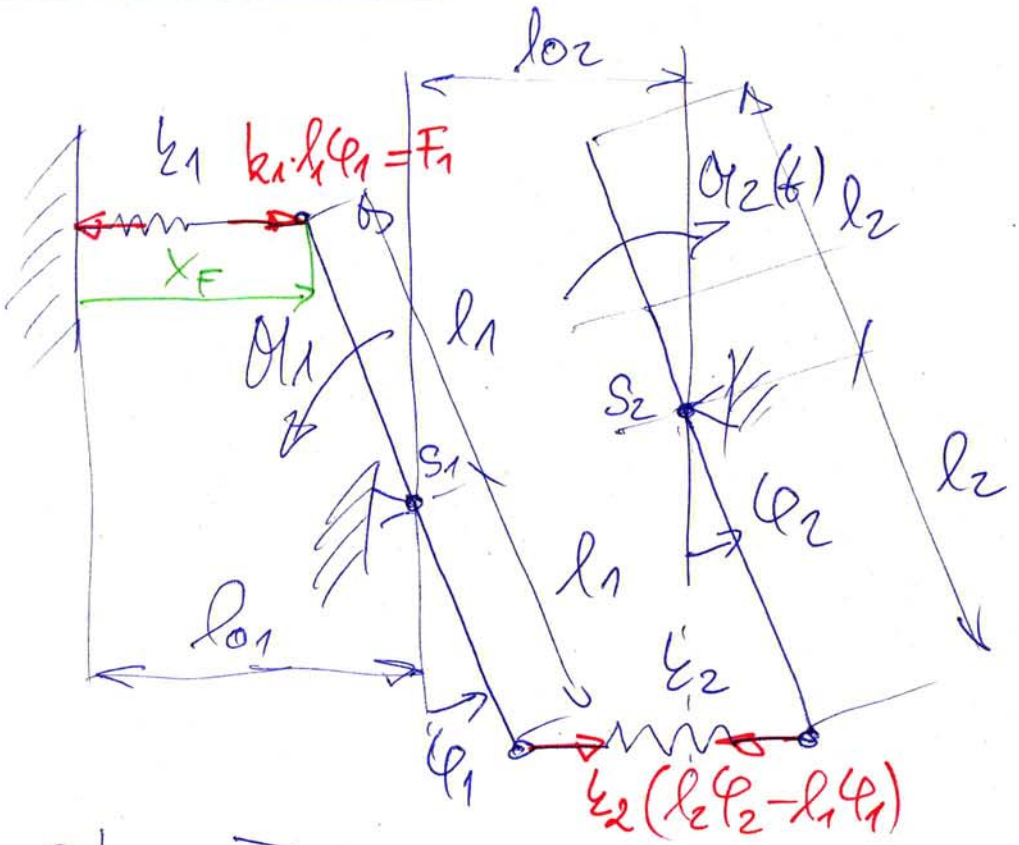


Predpoklad
$l_2\varphi_2 > l_1\varphi_1$

Obecně

$$E_{ki} = \frac{1}{2} m_i \left(\dot{x}_{si}^2 + \dot{y}_{si}^2\right) + \frac{1}{2} I_{si} \cdot \dot{\varphi}_i^2$$

$$E_k = \frac{1}{2} I_{1s1} \dot{\varphi}_1^2 + \frac{1}{2} I_{2s2} \dot{\varphi}_2^2$$

LR II

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}_i}\right) - \frac{\partial E_p}{\partial q_i} = Q_i \qquad i = 1, 2 \qquad q = \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix}$$

$i=1$

$$\frac{\partial E_k}{\partial \dot{\varphi}_1} = I_{S1}\dot{\varphi}_1$$

$$\frac{d}{dt}(\,) = I_{1S1}\ddot{\varphi}_1$$

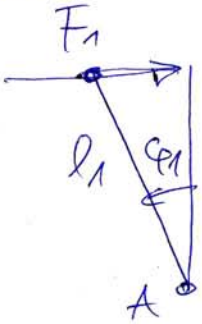
$$\frac{\partial E_k}{\partial \varphi_1} = 0$$

$i=2$

$$\frac{\partial E_k}{\partial \dot{\varphi}_2} = I_{2S2}\dot{\varphi}_2$$

$$\frac{d}{dt}(\,) = I_{2S2}\ddot{\varphi}_2$$

$$\frac{\partial E_k}{\partial \varphi_2} = 0$$



$$M_A = F_1 \cdot l_1 \cos\varphi_1$$

$\varphi_1 \ll 5^\circ$

$\Rightarrow \sin\varphi \doteq \varphi$

$\cos\varphi \doteq 1$

?

model kmitá

Zobecnělé síly

$$\sum_1 Q_i \delta q_i = -k_1 l_1 \varphi_1 \cdot l_1 \delta\varphi_1 + k_2(l_2\varphi_2 - l_1\varphi_1)\cdot l_1 \delta\varphi_1 - k_2(l_2\varphi_2 - l_1\varphi_1) l_2 \delta\varphi_2 + M_1\delta\varphi_1 - M_2\delta\varphi_2$$

$Q_1\delta\varphi_1 + Q_2\delta\varphi_2$

Virt. práce F_1

$$\sum_1 Q_i \delta q_i = F_1 \cdot \delta x_F$$

$$x_F = l_{01} - l_1 \cdot \sin\varphi_1$$

$$\delta x_F = -l_1 \cos\varphi_1 \delta\varphi_1$$

lze: $\delta x_F = -l_1 \cdot \delta\varphi_1$

$$\hookrightarrow = k_1 \cdot l_1 \varphi_1 \cdot (-l_1)\delta\varphi_1$$

Pohybové rovnice

$i=1$ $\quad I_{1S1}\ddot{\varphi}_1 = -\underbrace{\xi_1 l_1^2 \varphi_1 + \xi_2 (l_2\varphi_2 - l_1\varphi_1) l_1}_{Q_1} + M_1$

$i=2$ $\quad I_{2S2}\ddot{\varphi}_2 = -\underbrace{\xi_2 (l_2\varphi_2 - l_1\varphi_1) l_2}_{Q_2} - M_2(t)$

Maticový tvar

$$\begin{bmatrix} I_{1S1} & 0 \\ 0 & I_{2S2} \end{bmatrix} \begin{bmatrix} \ddot{\varphi}_1 \\ \ddot{\varphi}_2 \end{bmatrix} + \begin{bmatrix} \xi_1 l_1^2 + \xi_2 l_1^2 & -\xi_2 l_1 l_2 \\ -\xi_2 l_1 l_2 & \xi_2 l_2^2 \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \begin{bmatrix} M_1 \\ -M_2(t) \end{bmatrix}$$

$$\underline{M} \cdot \ddot{\underline{x}} + \underline{K} \cdot \underline{x} = \underline{f}$$

Frekv. rovnice

$$\det(\underline{K} - \lambda \underline{M}) = 0$$

$$\det \begin{bmatrix} \xi_1 l_1^2 + l_1^2 \xi_2 - \lambda I_{1S1} & -l_1 l_2 \xi_2 \\ -l_1 l_2 \xi_2 & \xi_2 l_2^2 - \lambda I_{2S2} \end{bmatrix} = 0$$

$$\Downarrow$$

$$\lambda_1, \lambda_2 \qquad \Omega_1 = \sqrt{\lambda_1}, \ \Omega_2 = \sqrt{\lambda_2}$$

Vlastní tvary kmitání

$$(\underline{K} - \lambda_i \underline{M})\,\underline{u}_i = \underline{0}$$

$\boxed{i=1}$ $(\Omega_1) \rightarrow \underline{u}_{j1}$ $j=1,2$

ω_1 ω_2

$$\underline{U} = [\underline{u}_1, \underline{u}_2] \leftarrow \underline{\varphi}$$

$$\underline{U} = \left[\begin{array}{c|c} u_{11} & u_{12} \\ u_{21} & u_{22} \end{array}\right] \begin{array}{l} \leftarrow \varphi_1 \\ \leftarrow \varphi_2 \end{array}$$

$$(k_1 l_1^2 + l_1^2 \xi_2 - \lambda_1 I_{1S1})\,u_{11} - l_1 l_2 \xi_2 u_{21} = 0$$

$$-l_1 l_2 \xi_2 u_{11} + (\xi_2 l_2^2 - \lambda_1 I_{2S2})\,u_{21} = 0$$

sečtyřmi postupem pro $\boxed{i=2}$ určíme u_{22}

Normování pro $u_{1j} = 1$

$\boxed{i=1}$ $u_{11} = 1$

2. rovnice

$$-l_1 l_2 \xi_2 \cdot 1 + (\xi_2 l_2^2 - \lambda_1 I_{2S2})\,u_{21} = 0$$

$$\boxed{u_{21} = \frac{l_1 l_2 \xi_2}{\xi_2 l_2^2 - \lambda_1 I_{2S2}}}$$

$$\underline{U} = \begin{bmatrix} 1 & 1 \\ u_{21} & u_{22} \end{bmatrix}$$

Matice dyn. tuhosti $(\underline{K} - \omega^2 \underline{M})$
↳ budicí frekvence

$$(\underline{K} - \omega^2 \underline{M})\,\underline{r} = \underline{f}_0$$

Soustavu budeme budit pouze $M_2(t)$

$$M_2(t) = M_{20} \cdot \sin(\omega t) \quad \Rightarrow \underline{f} = \begin{bmatrix} 0 \\ M_{20} \end{bmatrix} \sin(\omega t)$$

Matice dyn. poddajnosti $\underline{G} = (\underline{K} - \omega^2 \underline{M})^{-1}$

$$\underline{r} = \underbrace{(\underline{K} - \omega^2 \underline{M})^{-1}}_{\underline{Z}^{-1}} \underline{f}_0 = \underline{G}\,\underline{f}_0$$

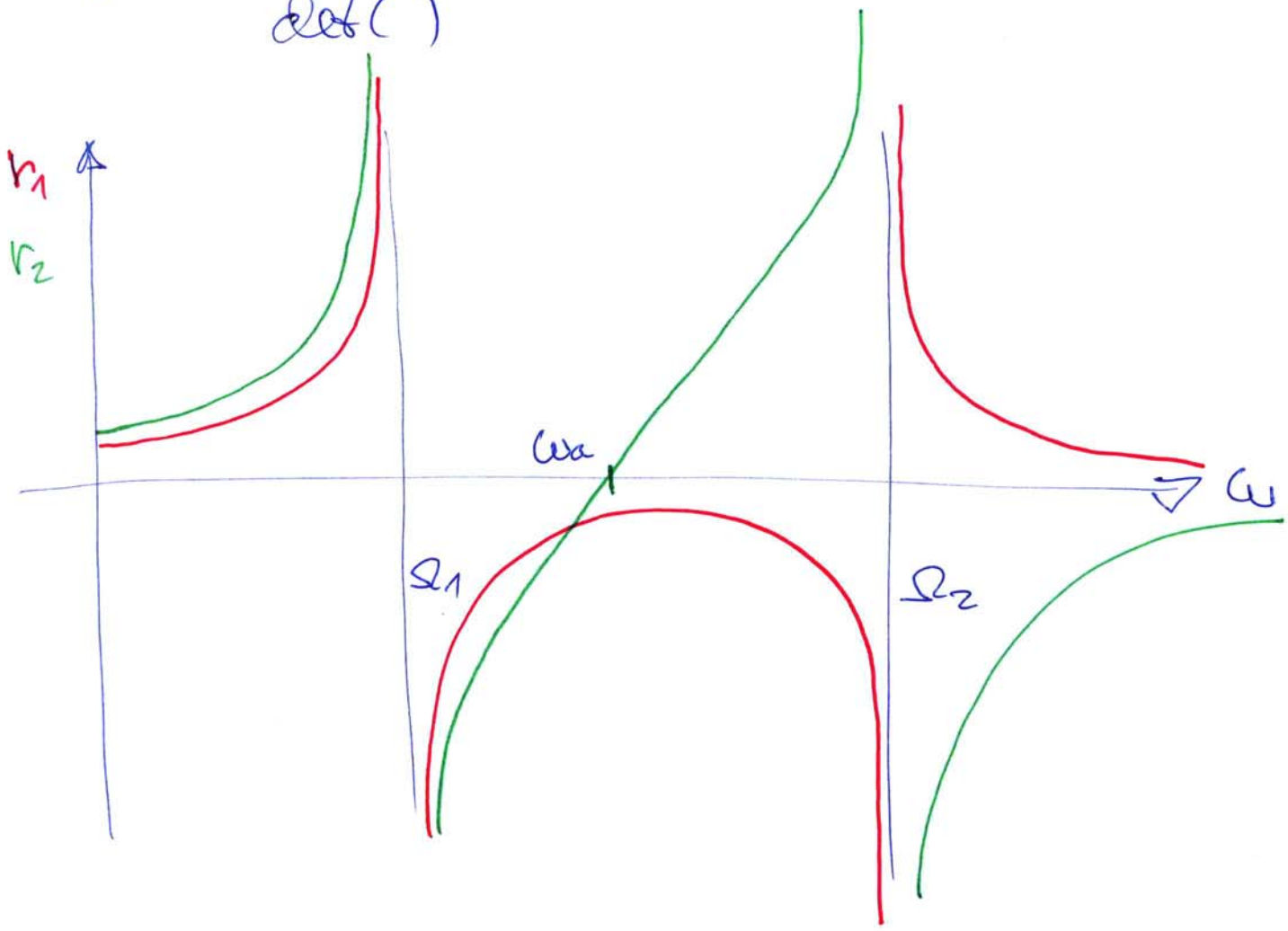
$$\underline{G} = \underline{Z}^{-1} = \frac{adj\,(\underline{Z})}{det\,(\underline{Z})} \quad , \quad \underline{G} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

$$\underline{r} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} 0 \\ M_{20} \end{bmatrix} = \begin{bmatrix} g_{12} M_{20} \\ g_{22} M_{20} \end{bmatrix}$$

$$\underline{G} = \left[\begin{array}{c|c} \dfrac{\xi_2 l_2^2 - \omega^2 I_{ss2}}{det(\,)} & \dfrac{l_1 l_2 \xi_2}{det(\,)} \\ \hline \dfrac{l_1 l_2 \xi_2}{det(\,)} & \dfrac{l_1^2(\xi_1 + \xi_2) - \omega^2 I_{ss1}}{det(\,)} \end{array}\right]$$

$$r_1 = \frac{l_1 l_2 k_2 \cdot \vartheta_{20}}{\det(\,)}$$

$$r_2 = \frac{[l_1^2(k_1+k_2) - \omega^2 I_{1S1}]\,\vartheta_{20}}{\det(\,)}$$



Antirisonanza

$$l_1^2(k_1+k_2) - \omega_a^2 I_{1S1} = 0$$

$$\omega_a^2 = \frac{l_1^2(k_1+k_2)}{I_{1S1}} \Rightarrow \boxed{\omega_a = \sqrt{\frac{l_1^2(k_1+k_2)}{I_{1S1}}}}$$